POWER MINIMIZATION ALGORITHM FOR HIGH-SPEED ACCESS OVER COPPER: MULTICHANNEL APPROACH

Ali Enteshari, Mohsen Kavehrad (FIEEE)

Center for Information and Communications Technology Research
The Pennsylvania State University, University Park, PA, 16802 USA

ABSTRACT

We present an algorithm to reduce the power consumption of single-carrier data transmission over twisted-pair copper cables for future IEEE networking physical connectivity standard, 40GBASE-T. High-complexity coding schemes employed for multi-gigabits transmission systems for higher reliability, e.g., system margin, is one of the major sources of power dissipation. The proposed method reduces the total power consumption by properly partitioning the channel into few sub-channels and using lower complexity coding schemes for the sub-channel with less insertion loss while the margin requirement is met over the entire channel.

Keywords: 40GBASE-T, single-carrier, multichannel.

1. INTRODUCTION

The single-carrier data transmission is traditionally implemented for high-speed data transmission over twisted-pair copper cables for Ethernet applications employing finite-length decision feedback equalizers. As the demand for higher speeds and faster interconnections within networks is explosively growing, data rates approach the Shannon capacity of the channel, and sophisticated coding schemes, e.g., turbo or LDPC coding, are required to fill the gap to capacity of single-carrier modulation. In 10GBASE-T application supporting a data rate of 10Gbps over CAT6 cable up to a distance of 100 meters, an LDPC assisted coded modulation achieving coding gains of about 9dB is implemented [1]. The encoder and decoder of this code consume roughly 25% of the total chip power. This is due to the fact that in single-carrier modulation, to obtain the required system margin (required SNR subtracted from achievable SNR) in the entire bandwidth, the coding circuitry has to run at a speed proportional to symbol rate (roughly twice the bandwidth for small bandwidth expansion).

For the next higher speed interconnections standard over copper cables, IEEE has announced their objectives for 40Gbps data transmissions [1]. It seems that following the legacy single-carrier implementation will put forward many serious challenges in terms of gate count, power consumption and latency. In this paper we propose an alternative architecture for IEEE 40GBASE-T application that can utilize channel bandwidth efficiently and as a result reduces the implementation complexity and power consumption. We conjecture an idea based on noise enhancement in channel equalization and sub-optimality of coded-modulation of single-carrier data transmission over highly dispersive channels. These channels are equalized at the receiver and coding is applied to improve the signal-to-noise ratio (SNR) degraded at the output of equalizer due to noise enhancement [2]. However, if we divide the bandwidth into a few channels, the noise enhancement in each sub-channel is less than the original channel, simply because there is less fluctuation of insertion loss when we look at it over shorter frequency spans.

This paper is organized as listed below. Following with the introduction section, a brief review on multi-channel communications theoretical and practical capacity bounds are presented in section 2. The optimal channel partitioning for power minimization is introduced in section 3. Simulation results for a multichannel system to transmit 40Gbps over 50m cable are presented in section 4, and we finish with our summary and conclusions, in the last section.

2. A BRIEF REVIEW

The cable channel is a highly dispersive communications medium. The insertion loss (attenuation) of the cable, which is a measure of decrease in signal strength along the length of the transmission line, depends on frequency, the diameter of copper wire, susceptivity and permittivity of isolator between wires, and many other factors. The capacity bounds of copper cables are solely determined by the insertion loss if Gaussian white noise assumption holds true and other interferer signals, e.g. far-end (FEXT) and near-end (NEXT) crosstalks, are negligible. The transfer function of 50m CAT-7A cable is shown in Fig 1.

In [3], it is shown that in the case of strictly monotonous decreasing channel attenuation, a constant power density in the first Nyquist set of frequencies $f \in [-1/2T, 1/2T]$ is optimum, where $T = 1/2W$ is the symbol period. In this case, the single-carrier bound, as a good performance measure for a channel impaired by background noise and intersymbol interference, can be defined as [4]:

$$C_{\mathrm{SC}} = W \log_2 \exp\left(\frac{1}{W}\int_0^W \ln\left(1 + \frac{\mathrm{SNR}(f)}{\Gamma}\right) df\right) \quad (1)$$

Literally, this limit is related to the so-called Salz SNR, which is often used in practical system implementations to estimate the system noise margin. In fact, this bound indicates the ultimate throughput of a real implementation of a system with finite coding gain and signal processing for any communication medium. One such implementation is the minimum mean-squared error decision feedback equalizer (MMSE-DFE) [2], or Tomlinson-Harashima Precoding (THP) [5].

In ultra high-speed applications where the trade-offs of power consumption, implementation complexity, and reliability are dramatically challenging, it is of considerable importance to study the problem of input symbol rate optimization under practical realizability constraints. Here, we are interested in achieving a fixed target bit rate while keeping the system margin as large as possible. This can be achieved by performance analysis of the coded system and link budget analysis for decision feedback equalizer (DFE) implementation. A specific reliability level, probability of error, is chosen and we seek to maximize the system margin to account for unforeseen sources of performance degradation. The system margin can be defined as [6][7]:

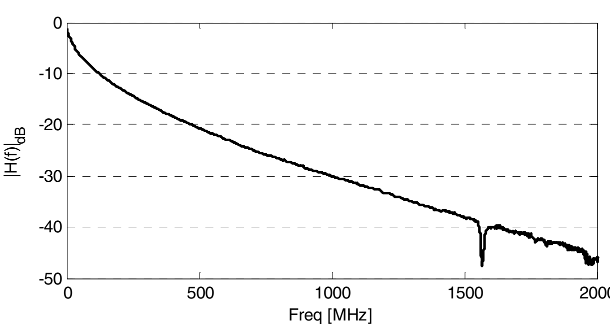

Fig. 1 Transfer function of 50m CAT-7A cable.

$$\gamma_m = \frac{3\mathrm{SNR}_{\mathrm{norm}} d^2_{\min}(\Lambda)}{\left(Q^{-1}\left(p_1 / K_{\min}\right)\right)^2} \quad (2)$$

where $K_{\min}$ is the multiplicity of codewords with minimum weight, $\gamma_c(\Lambda)$ is the *nominal coding gain* associated with set partitioning, $\gamma_s(\Lambda)$ is the *shaping gain*, and γ_m is the desired system margin. $\mathrm{SNR}_{\mathrm{norm}}$ is the normalized SNR and signifies how far the system is operating from the Shannon limit (the *gap to capacity*) [7].

Maximizing the data rate, for a set of parallel channels when the symbol rate is fixed, requires maximization of the achievable $C = \sum_n c_n$ over $\mathcal{E}_n$, the average power of each sub-channel. This is summarized as the following maximization problem, where $\mathrm{H}(f_n)$ represents the *n*th sub-channel transfer function of *k*-th channel.

$$\lim_{N\to\infty}\begin{pmatrix} \underset{\mathcal{E}_n}{\text{maximize}} \; W\sum_{n=1}^{N}\log_2\left(1 + \frac{\mathcal{E}_n \left|\mathrm{H}(f_n)\right|^2}{\Gamma N_n}\right) \\ \text{subject to} \sum_{n=1}^{N}\mathcal{E}_n = N\mathcal{E}_x \end{pmatrix} \quad (3)$$

where $\mathcal{E}_x$ is the average power of sub-channels. A natural solution for this optimization problem is to use Lagrange multipliers [8]. In this paper, we refer to the maximum value of this function, denoted by C_{WF}, as the *water-filling bound*.

The main idea in proposing a multi-channel approach is based on an important observation about this equation. Discrete multi tone (DMT) is an efficient implementation to achieve water-filling bound. At the receiver, a one-tap equalizer $W_n = 1/|\mathrm{H}(f_n)|$ is used at each sub-band to compensate for the channel attenuation. Because it is a single tap and cannot change the SNR, there is no noise enhancement due to this equalization, unlike other linear or nonlinear time-domain equalizers. When the number of sub-channels is not large enough, transmission on each sub-channel can be treated as regular single-carrier communication, but the noise enhancement in each sub-channel is less than when the entire bandwidth is used for single-carrier data transmission.

3. OPTIMAL MULTICHANNEL TRANSMISSION

We begin this section by the following remarks. Fig. 2 illustrates the system margin versus bandwidth for 40Gbps data transmission over 50m CAT-7A cable shown in Fig.

1. To achieve the required system reliability, with a 6dB margin, a coding scheme with 6dB gain and an optimum bandwidth around 1600 MHz are necessary [7]. A 6dB coding gain can be achieved by some complex multi-dimensional trellis coding or LDPC coded modulation. The coding and decoding schemes must operate at a speed proportional to bandwidth, which is 1600 MHz in this example.

If we divide this bandwidth, say up to two sub-bands, with the same average transmit power, we conjecture that the sub-channel with a less sever insertion loss would need a lower coding gain to achieve the requirements. This is somewhat similar to the argument that water-filling always outperforms over uniform power distribution.

In general, we divide up the bandwidth of W into N parallel sub-channels where w_n is the bandwidth of the *n*-th channel. Each sub-channel has its own dedicated equalizer. The capacity and system margin of the *n*-th channel are defined as [4]:

$$\begin{aligned} C_n &= w_n \gamma_\infty(w_n, g_n) \\ &= w_n \log_2 e^{\frac{1}{w_n}\int_0^{w_n} \ln\left(1+\frac{\mathrm{SNR}(f+\sum_1^{n-1} w_k)}{\Gamma(g_n)}\right) df} \end{aligned} \tag{4}$$

$$\gamma_n^m(w_n, g_n) = \frac{3 g_n \gamma_\infty(w_n, 1)}{2^{\frac{R_{bn}}{W}} \left[Q^{-1}(P_e / N_e)\right]} \tag{5}$$

where $R_n \le C_n$ is the data rate of the *n*-th sub-channel.

The total bit rate of R_b is distributed among these channels while we would like to satisfy the following objectives:

- bandwidth constraint: $\sum_n w_n = W$
- rate constraint: $\sum_n R_n = R_b$

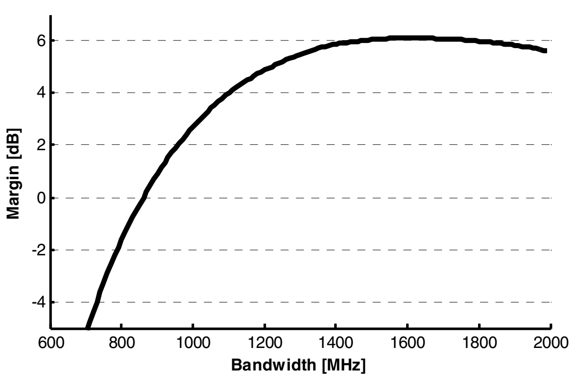

Fig. 2 System margin of 40GBASE-T system over 50m CAT-7A cable

- each sub-channel meets the required margin, i.e., $\gamma_n^m \ge \gamma_c$, $n = 1, \ldots, N$.
- coding gains constraint: $1 \le g_n \le g_c, n = 1, \ldots, N$

Our goal is to minimize the total power. A meaningful cost function in this regard can be expressed as

$$\phi(w_n, g_n) = k_1 w_n e^{k_2 g_n} \tag{6}$$

where we assumed exponential growth of coding and decoding scheme as the coding gain increases. We also assume that chip power and area are linearly scaled by frequency. Although we simply modeled this by a first-order approximation, more accurate and complex dependency can be incorporated into this model.

$$\begin{aligned} &\text{minimize} \;\; f_0(\boldsymbol{w}, \boldsymbol{g}) = \textstyle\sum_n \phi(w_n, g_n) \\ &\text{subject to} \;\; \textstyle\sum_n R_n = R_b \\ &\qquad\qquad \gamma_n(w_n, g_n) \ge \gamma_c, n = 1, \ldots, N \\ &\qquad\qquad 1 \le g_n \le g_c, n = 1, \ldots, N \end{aligned} \tag{7}$$

Regarding this optimization problem, we should note that although the objective function is separable, the margin constraint is not. This is due to the fact that the margin of n-th channel is the margin of sub-channel in $\left[\sum_1^{n-1} w_k, w_n\right]$ frequency range, which clearly depends on $w_k, k < n$. Besides, we show that the objective function and the constraints for this optimization problem are convex; therefore well developed algorithms to solve nonlinear convex optimization can be exploited here.

***Lemma* 1**: $\phi(w, g)$ is convex in both w and g variables.

proof: the convexity of $\phi(w, g)$ comes from the convexity of exponential and affine map in w.

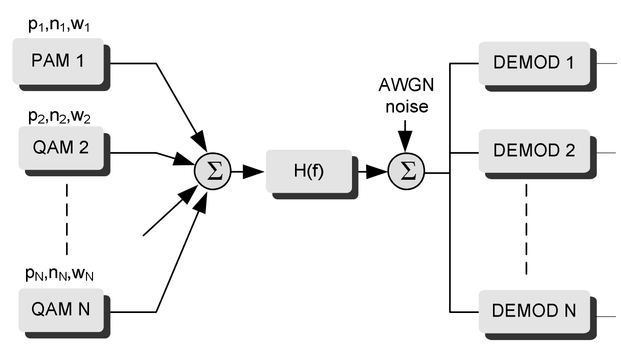

Fig. 3 Block diagram of multichannel system.

Lemma 2: The system margin of a bandlimited, strictly monotonous decreasing attenuation channel, $\gamma^m(w,g)$, is a concave function in both w and g variables.

proof: From (5), the concavity of $\gamma^m(w,g)$ in g is immediate. It is left to show that $\gamma_\infty(w,1)$ is concave in w. This can be shown in different ways. Here we present a simple algebraic proof. By checking (4) and noticing that logarithm is a concave function and linear fractionals preserve concavity [9], the concavity of $\gamma_\infty(w,1)$ becomes apparent.

These two lemmas enable us to perform conventional convex optimization algorithm to obtain the parameters for multi-channel transmission.

4. SIMULATION RESULTS

For the sake of implementation ease, only two parallel channels are considered here. Moreover, we need to do some simplifications as explained in the following in order to make the problem tractable. An important question in solving this optimization problem is how to divide the total data rate and assign them to sub-channels. It makes more sense if we assign the higher data rate to a sub-channel that has less sever condition, i.e., lower insertion loss (like water-filling algorithm [8]). Furthermore, we try to obtain a coding scheme with lower gain and less complexity for the channel with higher bandwidth while satisfying the system margin, as it was explained in the last section. We propose to use 4-D trellis coded modulation. This code can achieve a coding gain of about 4-5dB at low to moderate complexity.

Suppose that the bandwidth of the fist channel is w_1. The symbol rate of this channel is $R_{s1} = 2w_1 / (1+\alpha)$, where α is the excess bandwidth. If r_{b1} represents the number of uncoded bits per 4-D symbol, and M is the number of PAM levels in constituent single dimension of this constellation (we assumed that the constellation is based on $\mathbb{Z}^4$ lattice or its variants), then the following should hold for the expanded signal constellation to accommodate one extra coded bit from Viterbi encoder,

$$M^4 \geq 2^{r_{b1}+1} \tag{8}$$

From the past experiences, $M = 16$ seems to be the maximum number of levels that could be accommodated with the current technological advances and innovations in designing analog-to-digital converters at the speeds we are interested. The quantization noise and jitter are the main limiting factors in designing mixed-signal circuits for higher number of levels. Therefore, we fix the number of PAM levels to $M = 16$. Thus, the corresponding data rate for the first sub-channel becomes

$$R_{b1} = r_{b1} R_{s1} = 30 w_1 / \left(1+\alpha\right)$$

The data rate of the second sub-channel obviously becomes $R_{b2} = 40 \times 10^9 - R_{b1}$. Now, the set of constraints is defined and we can solve the problem. To solve this optimization problem with convex equality and inequality constraints, we consider the *Interior Point* method [9]. The variables for this optimization are w_1, w_2 and g_1, while $g_2 = 6\text{dB}$ is fixed. Results of this optimization are presented in Table 1.

Table 1 Parameters of 2-channel implementation

	Channel 1	Channel 2
R_b (Gbps)	29.404	10.596
Bandwidth (MHz)	1058	585
Coding gain (dB)	3.24	6
Code class	4-D trellis	LDPC
PAM levels	16	9

As we predicted, the channel with less sever insertion loss carries a higher data rate over a higher band. However, the coding gain for this sub-channel is reduced and this contributes to a substantial power reduction.

We finish this section noting the following remarks. First, we did not consider a guard band between the two sub-channels. However, the inclusion of a guard band into this analysis will only impact the upper sub-channel which is assigned to a lower data rate and less complex system. Second, we could take the coding gain of the second sub-channel as another variable and perform the optimization procedure. Third, in the implementation of this 2-channel system, the number of ADC and DAC converters is doubled, but each one operates at a lower frequency which makes the implementation much easier, as we know that the complexity of mixed-signal circuits grows exponentially.

4. SUMMARY AND CONCLUSIONS

As the demand for high-speed interconnections in backbone Ethernet increases, the design constraints for an efficient implementation become more challenging. In this paper we presented a novel multi-channel data transmission system for future IEEE networking physical connectivity standard, 40GBASE-T. The underlying theory based on generalized multi-channel data transmission is described and the power minimization algorithm by proper code selection is presented. Numerical simulations and results confirm the possible power reduction of 40GBASE-T system by this method.

5. ACKNOLEDGMENT

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