Bit Error Probability of Trellis-Coded Quadrature Amplitude Modulation Over Cross-Coupled Multidimensional Channels

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Abstract—**Convolutionally encoded M-ary quadrature amplitude modulation (M-QAM) systems operated over multidimensional channels, for example dual-polarized radio systems, are considered in this paper. We have derived upper bounds on the average bit-error probability for 4-QAM (QPSK) with conventional convolutional coding by means of a truncated union bound technique and averaging over the cross-polarization interference by means of the method of moments. By modifying this technique, we have found approximate upper bounds on the average bit-error probability for bandwidth efficient trellis-coded QAM systems. This is an extension of our previous work [1] that was based on one dominating error event probability as a performance measure. Our evaluations seem to indicate that bandwidth efficient trellis-coded M-QAM schemes offer much larger coding gains in an interference environment, e.g., a cross-coupled interference channel, than in a Gaussian noise channel. In general, our findings point out that optimum codes for a Gaussian channel are not optimum when applied in an interference environment. We note that a rate 1/2 convolutional code for example, with a code memory greater than two, if applied to two of the bits in each signal point representation, can be utilized with a simple decoder to greatly improve the performance of a QAM signal in interference. Also, we have introduced a new concept referred to as *dual-channel polarization hopping* in this paper which can improve the system performance significantly for systems with nonsymmetrical interference.**

I. Introduction

THIS paper is a continuation of the work presented in [1] and completes the earlier work on multidimensional signaling over cross-coupled channels using convolutional coding combined with M-ary quadrature amplitude modulation (M-QAM). These techniques have received great attention recently because of their potential in improving the capacity and performance of digital communication systems [2]. Trellis-coded modulation schemes achieve coding gain without bandwidth expansion [3]. Alternatively, bandwidth can be reduced for the same error probability performance. These properties are achieved by using higher order alphabets in modulation and coordinating the coding and modulation. Conventional bandwidth-expanding coded systems with independent coding and modulation [4], [5] will also be considered in this paper. In [1], all conclusions were drawn using only dominating error event probability calculations. In this work the previous conclusions are further supported by bit error probability evaluations based on all the contributing error events.

As shown in Fig. 1, four synchronous bit streams representing the in-phase and quadrature components of two M-QAM signals are convolutionally encoded. Encoding is applied to certain bits. The coded and uncoded bits are then mapped on to symbols of two QAM signals and then modulated. The modulated signals are polarized and transmitted over a flat-fading channel with circularly symmetric cross-coupling components. Due to fading and other impairments, the transmitted signals are cross-coupled and received in Gaussian noise. The received QAM symbols are demodulated and decoded via a Viterbi decoder. The receiver is optimized for the interference free case. A simple cross-polarization interference compensator [6] is used optionally to ease the detection operation. An optional interleaver/deinterleaver is also assumed to be used.

In our earlier work we found minimum distance and next to the minimum distance average error event probabilities for various cases where the convolutional coding was either applied to one or two bit streams. Coded bit streams were assumed to belong to one of the QAM signals or both. Performance was evaluated with and without a simple cross-coupling interference compensator and the accuracy of the results were supported by means of some asymptotic tools. The main objective of this paper is to evaluate average bit error probability for the signals of [1] and to present results for some new codes. This is done by employing a truncated union bound to calculate an approximate upper bound on the average bit error probability. Again the method of moments [1], [7] is the basis for our average error probability computations. Trellis codes are found to exhibit excellent performance for significantly high cross-coupling parameter values. As in [1] we note that some trellis-coded QAM systems yield a larger coding gain in interference and Gaussian noise than in the latter alone.

Almost all the published works on trellis-coded modulation schemes assume an ideal Gaussian channel. Some related work on nonlinear satellite channels is presented in [8]. Furthermore, the asymptotic coding gains are based on minimum Euclidean distance. Upper bounds on the error probability for certain classes of trellis-coded modulations are derived in [9] for the Gaussian channel. Bit error probability for constant amplitude digital modulation schemes consisting of combined convolutional codes and continuous phase modulation are presented in [10] for the Gaussian channel. Event error probability for an interference channel has been considered in [1] and also for some schemes in [11]. The bit error probability for conventional bandwidth expanding coding has been evaluated in [12] for cochannel interference.

Following the introduction, in Section II we describe the system model and theory briefly. In Section III, coded binary phase shift keying (BPSK) and quadrature phase shift keying (QPSK) are examined in a dual-polarized channel with and without an interference compensator. Trellis-coded M-QAM ($M > 4$) signals are studied and compared for a multidimensional channel in Section IV. Dual-channel polarization hopping, a new technique that results in diversity gains when applied to coded cross-coupled channels is presented in Section V. Our extensive numerical results for QPSK, 16-

Paper approved by the Editor for Modulation Theory and Nonlinear Channels of the IEEE Communications Society. Manuscript received March 24, 1986; revised September 25, 1986. This paper was presented in part at GLOBECOM'86, Houston, TX, December 1986.

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IEEE Log Number 8613150.

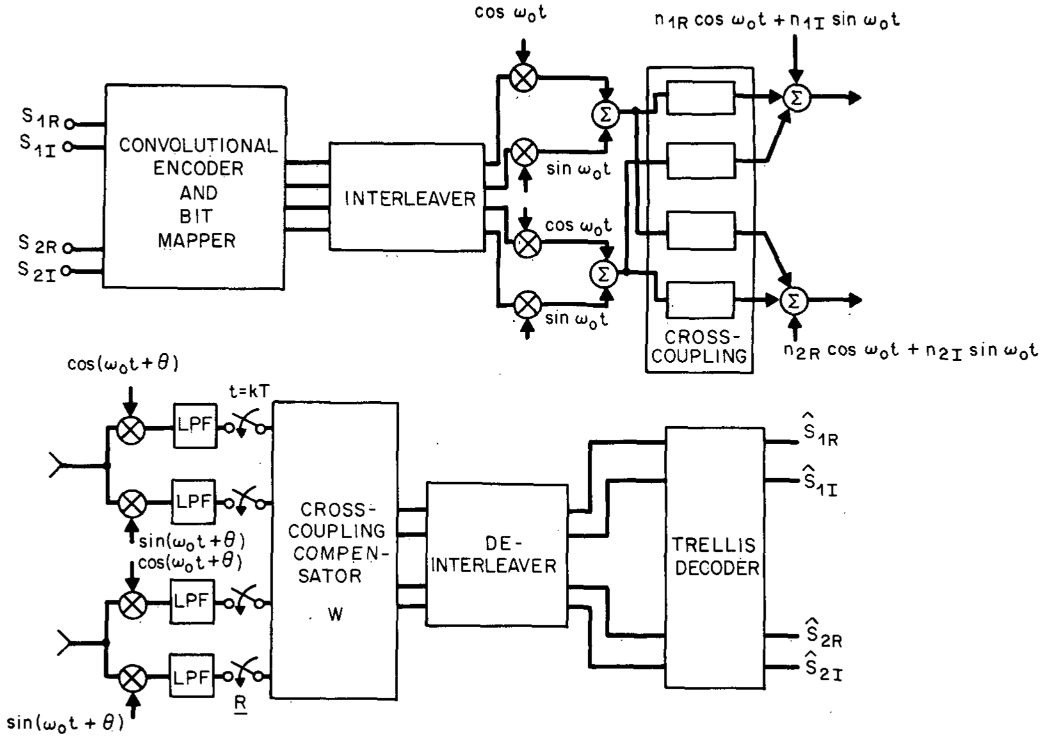

Fig. 1. System model.

QAM, 8-AMPM, 64-QAM, 32-AMPM, and QPSK with polarization hopping are included in Section VI. Finally, a summary and conclusions are presented in Section VII.

II. Channel Model and Theory for Error Event Probabilities

The signal transmission model is identical to that of [1] and we only review it here very briefly. The transmitted signals, $S_i(t)$, consist of two orthogonal M-QAM signals occupying the same bandwidth and using the same center frequency by means of dual-polarization. The baseband components of $S_i(t)$, $i = 1, 2$, are denoted by S_{iR}, S_{iI} since they represent real and imaginary parts of the complex envelope of these signals, namely,

$$\tilde{S}_i(t)=\sum_{k=0}^{\infty} \tilde{\alpha}_{ik}\tilde{h}(t-kT) \qquad i=1, 2 \tag{1}$$

where $\tilde{a}_{ik}$ represents the complex data symbols with real and imaginary parts δ_{ik} and β_{ik} taking values from the set $\{\pm C, \pm 3C, \cdots, \pm(L-1)C\}$ with $M = L^2$, the number of signal points in the QAM signal space. Also, $\tilde{h}(t)$ is the complex, low-pass equivalent impulse response of the system. Throughout the rest of this paper we assume $C = 1$. As in [1] we use the generalized matrix notations such that the sampled received signal is $\boldsymbol{R} = \boldsymbol{DH} + \boldsymbol{N}$ where $\boldsymbol{R} = [r_{1R}, r_{1I}, r_{2R}, r_{2I}]$ and $\boldsymbol{D} = [\delta_{1k}, \beta_{1k}, \delta_{2k}, \beta_{2k}]$ and the additive noise vector $\boldsymbol{N} = [n_{1R}, n_{1I}, n_{2R}, n_{2I}]$. The indexes R and I stand for real and imaginary parts of a complex quantity. For simplicity, the sampling index was dropped. The channel matrix $\boldsymbol{H}$ as in [1] is taken to be circularly symmetric, as follows

$$\boldsymbol{H}=\begin{bmatrix} 1 & 0 & \xi_2\cos\phi_2 & \xi_2\sin\phi_2 \\ 0 & 1 & -\xi_2\sin\phi_2 & \xi_2\cos\phi_2 \\ \xi_1\cos\phi_1 & \xi_1\sin\phi_1 & 1 & 0 \\ -\xi_1\sin\phi_1 & \xi_1\cos\phi_1 & 0 & 1 \end{bmatrix}. \tag{2}$$

Discussions on the validity of this model can be found in [6]. In (2), ξ_2 is the coupling coefficient from $S_1(t)$ to $S_2(t)$ and ϕ_2 is a random variable, uniformly distributed in $[0, 2\pi]$. Also, (ξ_1, ϕ_1) represents the coupling from $S_2(t)$ to $S_1(t)$. As an optional compensator the simple "diagonalizer" of [6] is used. The compensator forces all the cross-polarization coupled components in the overall channel matrix to zero. A possible model for this compensator is

$$W=\begin{bmatrix} 1 & 0 & -\xi_2\cos\phi_2 & -\xi_2\sin\phi_2 \\ 0 & 1 & \xi_2\sin\phi_2 & -\xi_2\cos\phi_2 \\ -\xi_1\cos\phi_1 & -\xi_1\sin\phi_1 & 1 & 0 \\ \xi_1\sin\phi_1 & -\xi_1\cos\phi_1 & 0 & 1 \end{bmatrix}. \tag{3}$$

In the case of using the compensator, the signal values for data detection can be represented by $\boldsymbol{R} = \boldsymbol{DHW} + \boldsymbol{NW}$. The system performance was readily evaluated in [1]. Note that the compensator can be modified to remove all interference (see [6]), however, in this work, as in [1], we purposely leave the residual cross-rail interference in order to find out how well coding behaves.

Briefly, to get the error event probability with or without coding, if $\boldsymbol{R}$ is the received signal vector and $\boldsymbol{s}_i$ is transmitted, $\boldsymbol{s}_i$ and $\boldsymbol{s}_j$, $i \neq j$, are two competing signals. Throughout the paper all signal, interference, and noise "vectors" consist of strings of one-dimensional or two-dimensional time samples [1]. An error event occurs when $\|\boldsymbol{R}-\boldsymbol{s}_i\|^2 > \|\boldsymbol{R}-\boldsymbol{s}_j\|^2$ where $\|\cdot\|^2$ is the squared Euclidean distance. Now assume

$$\boldsymbol{R}=\boldsymbol{s}_i+\boldsymbol{N}+\boldsymbol{I}$$

where $\boldsymbol{N}$ is the Gaussian noise vector, $\boldsymbol{I}$ is the interference vector. Then, an error occurs when $x_s > d_{ij}/2 - \boldsymbol{I}\cdot\Delta\boldsymbol{s}/d_{ij}$ where $x_s = \boldsymbol{N}\cdot\Delta\boldsymbol{s}/d_{ij}$, $d_{ij}^2 = \|\Delta\boldsymbol{s}\|^2$ and $\Delta\boldsymbol{s} = \boldsymbol{s}_j - \boldsymbol{s}_i$. The components of the vector $\boldsymbol{N}$ are all Gaussian and independent with zero-mean and variance σ_n^2. Hence x_s is zero-mean, Gaussian with variance σ_n^2. Thus the probability of an error event conditioned on the interference vector $\boldsymbol{I}$ is

$$P(\text{error event}|\boldsymbol{I}) = Q\left(\frac{d_{ij}}{2\sigma_n} - \frac{\boldsymbol{I}\cdot\Delta\boldsymbol{s}}{\sigma_n d_{ij}}\right) \quad (4)$$

where $Q(x) = (1/\sqrt{2\pi})\int_x^\infty e^{-t^2/2}\,dt$ is the error function. For example, for an interference vector $\boldsymbol{I} = \{I_k\} = \boldsymbol{\gamma} = \{\gamma_k\}$, $k = 1, 2, 3$ and a signal difference vector $\Delta\boldsymbol{s} = \{a_k\}$, $k = 1, 2, 3$ (4) can be expressed as

$$P(\text{error event}|\boldsymbol{I}) = Q\left(\frac{d_{ij}}{2\sigma_n} - \frac{\gamma_1 a_1 + \gamma_2 a_2 + \gamma_3 a_3}{\sigma_n d_{ij}}\right) \quad (5)$$

where

$$d_{ij}^2 = a_1^2 + a_2^2 + a_3^2.$$

III. Coded BPSK and QPSK

Before we proceed with the calculation of the upper bound on the average bit error probability for the trellis coded QAM systems, we will solve the simpler problem for QPSK and BPSK. Here we can use conventional upper bounds for convolutional codes over a Gaussian channel [4], [5] and combine them with the conditional error event probabilities from [1] for coding both with and without interference compensation. An alternative approach is presented in [12].

A. Coding Without Compensation

First, we will deal with the case of convolutionally encoded QPSK and BPSK without interference compensation. The analysis is carried out for one of the two cross-polarized channels. For the other channel, the upper bound on the bit error probability is obtained by replacing ξ_1 by ξ_2. A possible application here is a satellite link. In the same spirit and using (29) of [1], the error event probability for QPSK is

$$P(\text{error event}|\boldsymbol{I}) = Q\left(d_{ij}\sqrt{\frac{r_c E_b}{2N_0}}(1-X)\right) \quad (6)$$

where E_b is the energy per information bit, N_0 is the one-sided spectral density of the Gaussian noise, r_c is the overall rate of the coded system and where

$$X = \frac{2\xi_1}{d_{ij}^2}\left(\cos\phi_1 \sum_k a_k\delta_{2k} - \sin\phi_1 \sum_k a_k\beta_{2k}\right). \quad (7)$$

Independent coding is applied to the inphase and the quadrature phase components of the signal. The a_k's are the components of $\Delta\boldsymbol{s}$ and the summation is over all a_k's. For this case, the Euclidean distance for the error event is $d_{ij}^2 = 4\cdot d_{ij}^H$ where d_{ij}^H is the Hamming distance [4] for the error event, i.e., the Hamming distance between the coded binary sequences corresponding to $\boldsymbol{s}_i$ and $\boldsymbol{s}_j$. For uncoded QPSK, $d_{ij}^2 = 4$. Throughout the paper we will use the term error event both for components in Euclidean space and in the Hamming sense. Our definition of an error event is the conventional one for convolutional codes and trellis codes [4], [5], [9], and [10]. Furthermore, for coded QPSK, all nonzero a_k's are equal to 2. This fact simplifies the considerations for the QPSK scheme. Thus, for all error events we have

$$X = \frac{\xi_1}{d_{ij}^H}\cdot\sum_k(\delta_{2k}\cos\phi_1 - \beta_{2k}\sin\phi_1) = \xi_1\cdot X' \quad (8)$$

where the summation is carried out over all d_{ij}^H nonzero components (a_k) of $\Delta\boldsymbol{s}$. The worst-case interference parameter combination of ϕ_1, δ_{2k} and β_{2k} yields the maximum X for a given value of ξ_1 as $X = \xi_1 X'_{\max}$ with $X'_{\max} = \sqrt{2}$ both for uncoded and coded (all error events) QPSK. For the BPSK case, the event error probability is given by (7) with $X = (2\xi_1/d_{ij}^2)\cos\phi_1\sum_k a_k\delta_{2k}$. The worst-case interference combination yields $X = \xi_1 X'_{\max}$ with $X'_{\max} = 1$ for all error events for BPSK. This indicates that BPSK is more robust against interference than QPSK, both with and without coding. See [1], where extensive comparisons where made based on $X'_{\max}$.

From texts on convolutional codes, e.g., [4], [5] it is known that a good upper bound on the bit error probability for coded BPSK (or QPSK) in Gaussian noise is given by the union bound (see (6.11), pp. 243–244 of [4] and pp. 313–314 of [5])

$$P_b < \frac{1}{b}\sum_{k=d_f}^{\infty} w_k Q\left(\sqrt{\frac{2kr_cE_b}{N_0}}\right) \quad (9)$$

where $r_c = b/(b+1)$ with b information bits per trellis branch [4] and where d_f is the free Hamming distance of the code and the infinite sum in (9) in turn can be upperbounded by a so called transfer function bound [4]. This yields a closed form expression without the infinite sum. The coefficients w_k, the weight distribution of the code, are given by the code transfer function. In this study we are primarily interested in small or intermediate values of the average bit error probability P_b, say $P_b \leq 10^{-4}$. For this case, only the first few terms of the sum in (9) contribute to the bound. Throughout this paper we will evaluate the upper bounds by using a truncated version of (9). For a high E_b/N_0, the contributions for increasing values of k are smaller and smaller. We found empirically, that 5 to 10 terms in (9) are quite adequate for $P_b \leq 10^{-4}$. The rate 1/2 codes considered in this paper are given in Table I. The coefficients w_k for these codes are given in Table II. For further details, see [4], [5]. By combining (6) and (9), a truncated upper bound or the bit error probability conditioned on the interference can be calculated for any interference parameter combination. The conditional probability is

$$P_{b|I} \leq \frac{1}{b}\sum_{k=d_f}^{d_f+d_T} w_k Q\left(\sqrt{\frac{2kr_cE_b}{N_0}}(1-X_k)\right). \quad (10)$$

The summation in (9) is truncated to d_T terms. Furthermore, $X_k = (\xi_1/k)\cdot\Sigma_m(\delta_{2m}\cos\phi_1 - \beta_{2m}\sin\phi_1)$ where the summation is over all $k = d_{ij}^H$ terms corresponding to nonzero a_m's. By averaging over each of the d_T terms in (10) by means of the moment method, the truncated upper bound on the average bit error probability $\bar{P}_b$ is obtained. From the above the corresponding results for BPSK can easily be obtained.

B. Coding with Compensation

We will now combine the truncated union bound technique from Section III-A with error event probabilities for coding with interference compensation of the type discussed in Section II and in [1], [6]. Error event probability conditioned on interference for QPSK with independent coding on each of its two rails in a QPSK constellation is [1]

$$P(\text{error event}|\boldsymbol{I}) = Q\left(\sqrt{\frac{r_cE_b}{2N_0}}\cdot\frac{d_{ij}}{\sqrt{1+\xi_1^2}}\cdot(1-X)\right) \quad (11)$$

with

$$X = \xi_1\xi_2\left(\cos\phi - \frac{2}{d_{ij}^2}\sin\phi\sum_k a_k\beta_{1k}\right) \quad (12)$$

and $\phi = \phi_1 + \phi_2$. The conditional error event probability when the two QPSK rails are transmitted over *different*

TABLE I
RATE 1/2 CONVOLUTIONAL CODES FOR VITERBI DECODING WITH 2^v STATES. THE $v = 2$ ENCODER IS SHOWN IN FIG. 2 AND THE $v = 5$ ENCODER IN FIG. 3

ν	Code Connections
2	111,101
3	1111,1101
4	11101,10011
5	111101,101011
6	1111001,1011011

TABLE II
WEIGHTS FOR CALCULATING THE BIT ERROR PROBABILITY FOR THE RATE 1/2 CODES IN TABLE I. THE DATA ARE TAKEN FROM [5]

ν	d	w_d	w_{d+1}	w_{d+2}	w_{d+3}	w_{d+4}	w_{d+5}	w_{d+6}	w_{d+7}	w_{d+8}	w_{d+9}
2	5	1	4	12	32	80	192	448	1024	2304	5120
3	6	2	7	18	49	130	333	836	2069	5060	12255
4	7	4	12	20	72	225	500	1324	3680	8967	22270
5	8	2	36	32	62	332	701	2342	5503	12506	36234
6	10	36	0	211	0	1404	0	11633	0	76628	0

constellations (the S_{1R}, S_{2I} case in [1]) is

$$P(\text{error event}\,|\,\boldsymbol{I}) = Q\left(\sqrt{\frac{r_cE_b}{2N_0}}\cdot\frac{d_{ij}}{\sqrt{1+\frac{1}{2}(\xi_1^2+\xi_2^2)}}(1-X)\right) \quad (13)$$

with X given by (12). We will not further consider this case. Equation (11) is obtained from (13) with $\xi_2 = \xi_1$. For uncoded QPSK, the worst-case interference is obtained from (11) with $d_{ij}^2 = 4$, $a_1 = 2$, $\beta_{11} = -2$, $k = 1$ and $\phi = \pi/4$ yielding $X_{\max} = \xi_1\xi_2X'_{\max}$ and $X'_{\max} = \sqrt{2}$. For the coded case, the worst-case interference is also easily obtained from (11). For this case $d_{ij}^2 = 4\cdot d_{ij}^H$. Thus, for any error event, for the coded case we have $X'_{\max} = \sqrt{2}$. Thus we can conclude that the coding gain in Gaussian noise for very high signal-to-noise ratios is preserved for the interference case as well. We can expect larger coding gains at intermediate signal-to-noise ratios, as we will see in the Numerical Results section. The truncated upper bound on the average bit error probability for QPSK with cancellation and coding can now be obtained by combining (11), (13), and (9) and performing the averaging over the interference by means of the moment method. BPSK after cancellation is a trivial problem of no interest here since all the interference is compensated and only Gaussian noise remains.

IV. TRELLIS-CODED QAM

For trellis-coded modulations [3] with Viterbi decoding there are no general upper bounds on the average bit error probability available in the literature, not even for the Gaussian channel. Bounds for some special cases such as, e.g., trellis-coded M-ary phase shift keying (MPSK) are given in [9] for the ideal Gaussian channel. In this paper we will derive approximate upper bounds for a special class of trellis-coded QAM schemes. We will demonstrate that the coding gains indicated by the dominating error event probability (averaged over interference) considered in [1] also hold for the average bit error probability. The class of trellis-coded QAM schemes considered here consists of 2^m-ary QAM schemes with rate 1/2 coding on 2 bits yielding an overall rate of $(m-1)/m$. Fig. 2 shows a 16-QAM signal with rate 3/4 coding and Fig. 3 shows a 64-QAM signal with rate 5/6 coding. For further details on this type of trellis-coded modulations, see [3], [1], [13]. As in [1] we are interested in comparing the 2^m-ary coded rate $(m-1)/m$ QAM scheme to, e.g., an uncoded $2^{(m-1)}$-AMPM system which is nonbandwidth expanding. For definition of 8-AMPM and 32-AMPM constellations, see [2], [3]. The general problem with the analysis of trellis-coded QAM is that the linear property of the convolutional code [4], [5] is not immediately carried over to the trellis-coded QAM scheme [3], [9]. One effect of this is that the error event structure in general depends on the specific transmitted information sequence. This fact considerably complicates the general problem of deriving an upper bound on the error probability. Averaging now also has to be performed over all possible transmitted sequences, as in [10]. Another complication is the averaging over the interference, as in Section III.

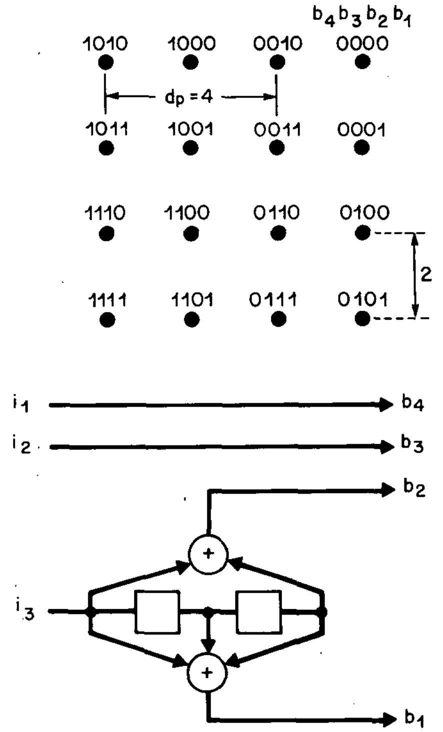

Fig. 2. Example of mapper and $v = 2$ convolutional encoder. The overall rate of the coded 16-QAM system is 3/4.

In this paper we are considering 2^m-ary QAM systems with interleaved natural binary mappers on each rail and with a rate 1/2 code applied to the least significant bit in each dimension, [13], [1]. This is a set partition mapping [3], [13]. For this mapper it is also easy to relate the Hamming distance structure of a well-known rate 1/2 convolutional code to dominating Euclidean distances for the trellis-coded modulation scheme. The maximum likelihood receiver consists of a Viterbi decoder with 2^v states, where v is the memory in the rate 1/2 convolutional code. There are $2^{(m-2)}$ transitions between the states (so-called parallel transitions). The detector makes the maximum likelihood decision in two steps [13]. One of the $2^{(m-2)}$ transitions is selected. Then a binary decision based on path metrics is made at each state (add, compare, and select) like in any conventional Viterbi detector for a binary rate 1/2 code [4], [5].

The error events can consist of short, 1 symbol errors, where the uncoded bits are in error. These are the parallel transition errors. The other types of error events are longer

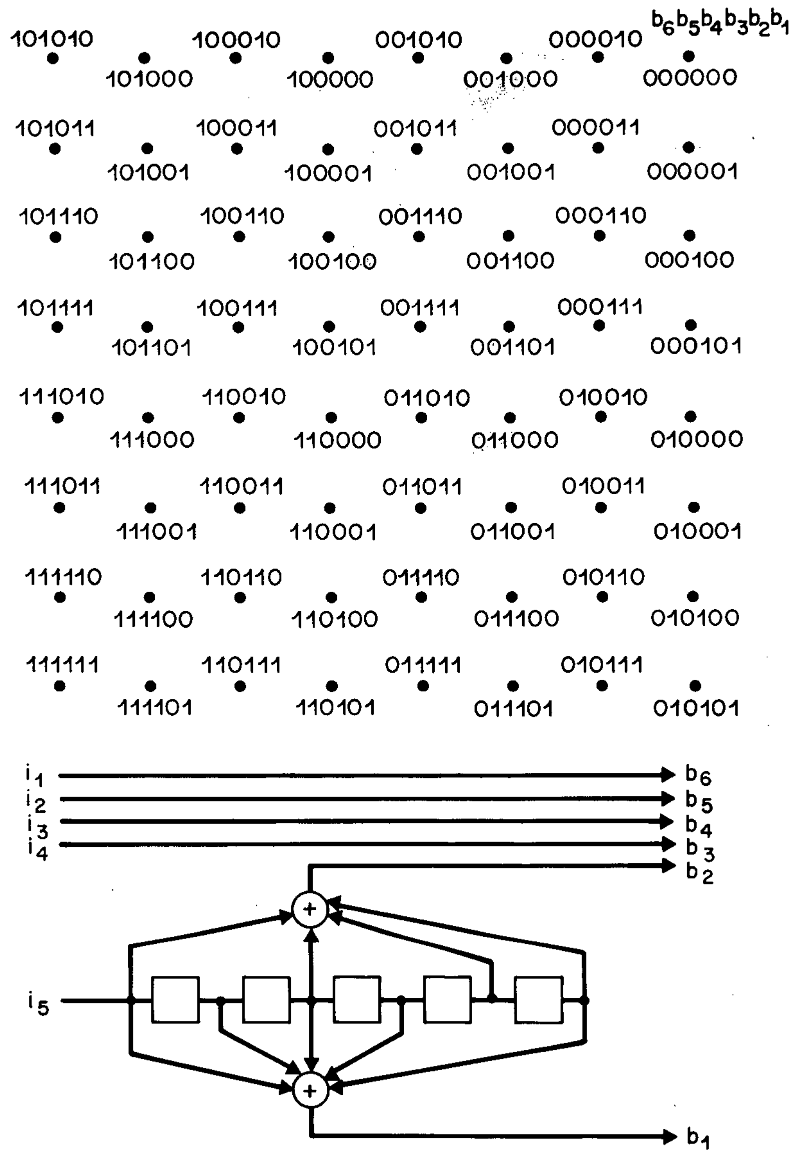

Fig. 3. Example of mapper and $\upsilon = 5$ convolutional encoder. The overall rate of the coded 64-QAM system is 5/6.

error events consisting of several symbols. This corresponds to a case that the Viterbi detector has chosen the wrong path through the trellis. All the error events now correspond to errors in the coded bits and sometimes also errors in the uncoded bits. It is the purpose of this paper to count the number of significant error events and evaluate the number of bit errors.

For Gaussian channels it is well-known that using trellis-coded QAM schemes with 2 coded bits only, the error probability performance for high channel signal-to-noise ratios is not improved by increasing the code memory beyond $\upsilon = 2$, [3]. The reason for this is that the minimum Euclidean distance is given by the parallel transition distance, which is not affected by the code memory. However, in interference channels this is no longer true, [1]. Therefore, we consider rate 1/2 codes of longer memory than 2.

A. Coding Without Canceller

First we consider trellis-coded QAM without interference cancellation. The error event probability for all error events for the trellis-coded modulation schemes considered here conditioned on interference, have been calculated in [1]. This will be denoted $P(E_{ij}|\boldsymbol{I})$ throughout the paper, where E_{ij} is the error event, index i refers to the transmitted signal and j refers to the received one. Based on union bound, the first event error probability [4-5] conditioned on $\boldsymbol{I}$ can be upper bounded by

$$P(E|\boldsymbol{I}) \leq \sum_i p_i \sum_{\substack{j \\ i \neq j}} P(E_{ij}|\boldsymbol{I}) \tag{14}$$

where the outer sum is an average over all the *transmitted* sequences with probability density p_i and the inner sum is a bound on the first error event probability conditioned on the transmitted sequence i and the interference. Note that the interference $\boldsymbol{I}$ is independent of the transmitted information sequence [1]. In Section III, we did not have to average over the transmitted sequences since the same set of error events occur for all transmitted sequences. For any transmitted code sequence the convolutional code is linear and all calculations can without lack of generality be based on the transmitted, all zero, coded sequence. Although (14) seems formidable to evaluate, we can significantly simplify the task of calculating this bound by the following observations. The Euclidean distance for the dominating error events caused by the convolutionally coded part of the sequence can be directly related to the Hamming distance of the error events of the code. Any particular error event with any given Hamming distance can occur for any transmitted sequence of coded QAM-points. Thus all the code error events in the coded two least significant bits occur for all transmitted sequences of coded QAM-points. They do, however, occur more frequently for certain transmitted sequences than for others.

Before we rewrite (14) in a more convenient form, let us give some examples of error events for the $v = 2$, rate 3/4 coded QAM scheme in Fig. 2. Three types of error events can occur, namely one symbol error events caused by parallel transition errors and two different types of "longer" error events caused by the Viterbi detector choosing the wrong path through the trellis.

Some examples:

1) When the transmitted signal point 0000 is received as 1000, there is a parallel transition error event with the normalized parallel transition distance $d_{ij} = d_p = 4$. Equally likely is the parallel transition error 0100 when 0000 is transmitted. In both cases bit errors occur in the first 2 bits only, i.e., in the uncoded bits. Much less likely is the diagonal parallel transition error 1100, ($d_p = 4\sqrt{2}$).

2) The dominating "long" error event when a sequence of 0000 signal points is transmitted is 0011, 0001, 0011 which is of Hamming distance 5 from the transmitted coded binary sequence (all zero) in the last two bits in each signal point and of distance $d_{ij}^2 = 20$ measured in squared normalized Euclidean distance [1]. In this particular example there are no bit errors in the first two bits. All errors occur in the last two coded bits. This is of course true for all error events with $d_{ij}^2 = 20$ conditioned on the transmitted sequence of signal points 0000.

3) For a more general sequence of transmitted signal points, the same error event as in example (2) can always occur but also errors in both the coded and the uncoded bits. For example, if 0011, 0011, 0011 is the transmitted sequence, the received sequence 0000, 0010, 0000 corresponds to an event with $d_{ij}^2 = 20$ and only errors in the last 2 bits like before, while, e.g., the received sequences 1100, 0010, 0000; 1000, 0010, 0000; 0100, 0010, 0000; etc. are examples of sequences of distance $d_{ij}^2 = 20$ from the transmitted sequence where the Hamming distance from the transmitted binary *coded* sequence is 5 (last 2 bits corresponding to the signal points) and where errors occur also in the uncoded first two bits of the binary representation of the signal points.

Based on the above examples we can note that the coded error event 11, 01, 11 (last 2 bits only for each signal point) occurs for any transmitted sequence of signal points. However, it occurs more than once for some combinations. The same holds for all other long error events. We can now start rewriting (14) and expressing it in error events with a certain Hamming weight (two last bits of the coded sequences), thus utilizing the knowledge we already have about the conventional transfer function bound for the convolutional code and thus the Hamming distance error structure of the code. The error event probability conditioned on interference for the coded QAM system is given by (28) and (29) in [1]. Thus

$$P(\text{error event}\,|\,I) = Q\left(\sqrt{\frac{3r_cE_b\log_2(L)}{2N_0(L^2-1)}}\cdot d_{ij}(1-X)\right) \tag{15}$$

with $L^2 = 2^m$ and

$$X = \frac{2\xi_1}{d_{ij}^2}\left(\cos\phi_1\sum_k(a_k\delta_{2k}+b_k\beta_{2k}) - \sin\phi_1\sum_k(a_k\beta_{2k}-b_k\delta_{2k})\right). \tag{16}$$

For the error events in the two-dimensionally coded QAM system where the absolute value of the signal difference vector components $|a_k|$ and $|b_k|$ are at most equal to 2, the squared Euclidean distance is $d_{ij}^2 = 4d_{ij}^H = 4d$ where $d_{ij}^H = d$ is the Hamming distance between the binary coded sequences (last 2 bits of each signal point). Only this type of "long" error event will be considered for the bound. There are other "long" error events, but their contribution to the bound is negligible. The reason these events are discarded is that they have at least one large distance component of the order of at least the parallel transition distance yielding a total Euclidean distance that is large. Furthermore, these error events are less susceptible to the interference [1].

In the following, the event error probability in (15) is denoted $P(2\sqrt{d}|I)$ where $d_{ij} = 2\sqrt{d}$ and d is the Hamming distance. By counting all the significant contributing error events we can now formulate the *approximate* upper bound on the bit error probability in the coded information bit (the bit that is fed into the binary rate 1/2 code, see Figs. 2 and 3)

$$P'_{b|I} \lesssim \sum_{d=d_f}^{d_f+d_T} w_d\cdot(C_m)^d\cdot P(2\sqrt{d}\,|I) \tag{17}$$

where C_m is a constant depending on m. $P(2\sqrt{d}|I)$ is given by (15). For QAM constellations and even m, this constant is

$$C_m = (2^{m/2}-1)2^{(1-m/2)}. \tag{18}$$

This number C_m is the result of counting all the signal point error events with a given rate 1/2 code error event. For example, for the 11, 01, 11 code error event for 16-QAM, there are

$$\left(\frac{1+2+2+4}{4}\right)\times\left(\frac{1+1+2+2}{4}\right)\times\left(\frac{1+2+2+4}{4}\right)=\left(\frac{3}{2}\right)^5$$

contributions to the signal point error events. The denominator comes from averaging over all equally likely transmitted sequences. Fortunately, the constant in (18) can easily be generalized to longer error events and other signal constellations. We found in [1] that the particular structure of the long error event for a given Euclidean distance affects the conditional error probability $P(2\sqrt{d}|I)$ for the case of coding without compensation. In particular, two coded 10 or 01 sections (referring to the coded bits) are worse than one 11 section although their contribution to the distance is the same. Thus, it is necessary to differentiate between error events with the same distance d but different structures. For the $v = 2$ code, there is only one case, namely all error events begin and end with 11 and in between there are only 01 and 10 sections. This follows immediately from Fig. 6.10 and related text in [4]. All error events for the optimum $d_f = 5$, $v = 2$, rate 1/2 convolutional code have two 11 sections and the remaining $d - 4$ sections contributing to the distance for an error event of Hamming weight d are of type 01 or 10. The only two 11 sections in the error event occur at the beginning and end of the error event. For general rate 1/2 codes and in particular those with $v > 2$, sections of type 11 can also occur in the inner sections of an error event. For good codes, the first and the last section of an error event are always 11. Conventional transfer function bound techniques can be modified so that error events with a certain number of 11, 01, and 10 sections can be counted separately. This is required to make the analysis precise. However, as an indication of the performance, we can upper and lower bound the contributions for a given d by assuming that all the error events are of the worst (many 01 or 10 sections) or of the best (many 11 sections) type, respectively. The moment method can be used to evaluate the average of $P(2\sqrt{d}|I)$ for any particular error event structure.

We will now derive the contribution to the overall average bit error probability from the parallel transitions. For 16-QAM, the average number of dominating parallel transition errors per transmitted signal point is 2. Each error causes one bit error in 3 information bits. Thus the contribution to the bit error from a parallel transition error event conditioned on the interference is $(2/3)P(d_p|I)$ where $d_p^2 = 16$ is the Euclidean distance for the dominating error event (4, 0) or (0, 4). Equations (15) and (16) yield in the conditional error event probability. For 64-QAM there are an average of 3 dominating parallel transition error events per transmitted signal point and the average number of information bit errors per parallel transition error is 4/3. Each signal point carries 5 information bits. The corresponding numbers for 256-QAM and other schemes can easily be worked out. The approximate contribution to the conditional bit error probability is denoted $C_pP(d_p|I)$ where $C_p = 2/3$ for the 16-QAM and $C_P = 4/5$ for the 64-QAM schemes.

Finally, we need to evaluate the contribution from the long error events to the overall bit error probability. Equation (17) gives the bit error probability in the coded information bit. If the other most significant bits were error free, the overall contribution to the total average conditional bit error probability would be $1/(m - 1)$ times that of (17). Since the first $(m - 2)$ bits clearly are not error free all the time when long error events occur, this is a lower bound on such a contribution. Evaluation of the exact coefficient for the contribution from the most significant bits seems hard. We conjecture that (17) is an upper bound for the overall average conditional bit error probability without giving a formal proof. We will present a few observations to substantiate this claim. For the rate 3/4 16-QAM case, the average number of information bit errors per coded section of type 11 is 1/2 per 2 transmitted information bits in the uncoded part of a signal point. The averaging is being carried-out over all transmitted sequences and over all signal point sequence error events conditioned on any one particular code error event. The corresponding number for the 01 or 10 section is 1/4 bits. For the $v = 2$ code, the 11, 01, 11 Hamming error event dominates among the long error events. This event causes one information bit error in the third bit (coded bit) while the average number of bit errors among the two first information bits is $1/2 + 1/2 + 1/4 = 1.25$ or per bit $5/8 < 1$. Thus, our conjecture is pessimistic for this case. For the $d = 6$ error events, the average number of information bit errors in the third bit of the signal point per error event is 2 while $(1/2 + 1/2 + 2 \cdot 1/4)/2$ clearly is less. We obtain similar results for longer error events and other codes. We have not found any example where the conjecture does not hold. It is obvious from the discussion carried out above that the bit error probability varies somewhat with the particular bit in the mapper. For 16-QAM, each signal point carries 3 bits of information and in general each of these bits has a different average bit error probability. For the rate 5/6 64-QAM case (see Fig. 3), the average number of information bit errors among the 4 bits in the uncoded part corresponding to signal points where the code error event has a 11 section is 1. In this case the averaging has been performed over all transmitted sequences. The corresponding number for the 01 (and 10) section is 1/2. Thus the contribution to the overall average bit error probability is most likely upperbounded by (17) also for the 64-QAM case.

Thus, the overall conditional (with respect to the interference parameters) average (over the transmitted sequences and over the bits in each signal point mapping) bit error probability is approximately upper bounded according to

$$P_{b|I} \lesssim C_pP(d_p|I) + \sum_{d=d_f}^{d_f+d_T} w_d(C_m)^d P(2\sqrt{d}\,|I). \quad (19)$$

Finally, the overall average bit error probability is obtained by averaging each term in (19) over the interference by means of the moment method.

Thus, by calculating C_p, d_p and C_m for the trellis-coded QAM scheme and using the conventional weight structure w_d for the rate 1/2 convolutional code, we have obtained an approximate upper bound on the average bit error probability.

B. Coding with Canceller

It is now straightforward to obtain a corresponding approximate upper bound on the average bit error probability for the case of coding and cross-coupled interference cancellation. The formula in (19) still holds with different conditional event error probability expressions for $P(d_p|I)$ and $P(2\sqrt{d}|I)$. We now use the conditional error event probability from (40) and (41) in [1]. This is the expression for coding on two rails in *different* QAM constellations. (The interference is independent of the transmitted sequence [1].) We use this expression because it is easier to average over the interference in this case. With $\xi_1 = \xi_2$ we conjecture that there is no significant difference compared to coding on two rails in the same QAM constellation. The error event probability is

$$P(\text{error event}|I) = Q\left(\sqrt{\frac{3r_cE_b\log_2(L)}{2N_0(L^2-1)}} \cdot \frac{d_{ij}}{\sqrt{1+\xi_1^2}}(1-X)\right) \quad (20)$$

where

$$X = \xi_1^2\left(\cos\phi - \frac{2}{d_{ij}^2}\sin\phi\sum_k(a_k\beta_{1k} + b_k\delta_{2k})\right). \quad (21)$$

Contrary to the case in Section IV-A, the contributions to the overall bit error probability for a given long code error event with a given Hamming distance is not dependent on the particular structure of that code error event, i.e., the number of 11, 01, 10 sections. The one thing that matters is the Hamming distance $d_{ij}^H = d$ which is the number of interference terms (either β_{1k} or δ_{2k} or both) in the sum over k in (21). The sum is taken over all sections with $a_k, b_k \neq 0$. Compare the latter with the expression for the interference without cancellation, (16). In the latter case, the interference vector components corresponding to a_k, b_k are dependent, while in (21) they are independent. Thus the structure of the error event matters only for the case of no compensation. Note that the data sequence is given by δ_{1k}, β_{2k} while β_{1k}, δ_{2k} is interference, which is independent of the data symbols [1].

With the modifications of the error event probabilities, the overall average bit error probability conditioned on the interference is given by (19) and with (20), (21) and the averaging over the interference is obtained by using the moment method [1] on each term in (19). We note that the bound for the coded case with cancellation is tighter than the corresponding bound without cancellation. The reason is that the detailed error structure of the code error event is not required for coding and cancellation.

V. Dual-Channel Polarization Hopping for Coded QAM Schemes

In [1] we introduced a method to obtain diversity gains in trellis-coded QAM schemes transmitted over 2 cross-coupled channels with different interference coupling values, i.e., $\xi_1 \neq \xi_2$. No compensator is used. The principal idea is for each signal point to be sent as S_{1R} over one channel and S_{2I} over the other. Long error events will then have components from both the good (low ξ) and bad (high ξ) channels while for the no diversity case, all transmissions take place for the same ξ-value. This method of diversity only works for schemes with two-dimensional error events, like the long error events in trellis-coded QAM. It does not improve the performance for one-dimensional coding, like coded 8-AM on one rail or coded QPSK. Although this diversity option is only partially analyzed in [1], we could conclude based on worst-case analysis that some diversity gains are obtained when $\xi_2 < \xi_1$ while the worst case for $\xi_1 = \xi_2$ with diversity was worse than transmission without diversity with $\xi_1 = \xi_2$. An alternative and more widely applicable method of diversity is obtained by sending every other signal point over channel 1 and the remaining signal points over channel 2. This could be done in a 1, 2, 1, 2, 1, 2 $\cdots$ pattern or scrambled with 50 percent use of each channel. We will only consider the odd/even sample case here.

It follows immediately from the definition of the diversity transmission scheme that signal points at odd times are transmitted over channel 1 and at even times transmitted over channel 2. Then the error event probability is given by (15) with

$$X=\frac{2}{d_{ij}^2}\Bigg(\xi_1 \cos\phi_1 \sum_{\substack{k\\ \text{odd}}} (a_k\delta_{2k}+b_k\beta_{2k}) - \xi_1 \sin\phi_1 \sum_{\substack{k\\ \text{odd}}} (a_k\beta_{2k}-b_k\delta_{2k}) + \xi_2 \cos\phi_2 \sum_{\substack{k\\ \text{even}}} (a_k\delta_{2k}+b_k\beta_{2k}) - \xi_2 \sin\phi \sum_{\substack{k\\ \text{even}}} (a_k\beta_{2k}-b_k\delta_{2k}\Bigg) \tag{22}$$

We can see from (22) that for the extreme case of $\xi_2 \leq \xi_1$, $\xi_2 = 0$, the interference in a long error event extending over both odd and even times (k) is significantly reduced compared to no diversity and transmission over the ξ_1 channel. For the other extreme case, namely $\xi_2 = \xi_1$, the worst case with a long error event with "typical components" in both channels 1 and 2 will not change compared to transmission over one channel. We conjecture that averaging over interference for the two cases will yield approximately the same results. Further work on this issue is required. In this paper we will only analyze one special case of this time diversity, namely, QPSK and $\xi_2 = 0$. For this case, the error event probability of (6) applies with the summation in (7) taken only over odd values of k. We have approximately evaluated the average bit error probability for this case by using (10) and by discarding all the even positions in an error event of Hamming weight d. That is to say, an error event of Hamming weight d is represented by a sequence of d 1's and every 1 in an even position is dropped. Since only the number of 1's in the error event affects the final average error event probability for the case of no diversity, we did not have to account for the detailed structure of the error event. As was pointed out in connection with trellis-coded QAM, for a given Hamming weight d there are several different types of error events of different lengths consisting of a mixture of 0's and 1's. Dropping every even symbol does not necessarily give a Hamming weight of $d/2$ for an even d and $(d + 1)/2$ for an odd d. However, this is a reasonable first approximation for large d_f. To obtain a precise result, every error event of the code must be generated and (22) must be evaluated in detail. Approximately, however, half of the components in long "typical" error events do not contribute to (22). Both the approximate and the precise evaluation of $\bar{P}_b$ for the $\xi_2 = 0$ case can be done with the moment method described in [1]. The more general case where $\xi_2 \neq 0$ requires further work.

For the other extreme case, diversity and $\xi_1 = \xi_2$ we again conjecture that the one channel evaluation is representative, both in terms of average and worst-case results. It will be shown in the next section, that significant diversity gains can be obtained with this scheme for QPSK with rate 1/2 coding.

VI. Numerical Results

The average bit error probability as a function of E_b/N_0 and interference level 20 $\log_{10}$ (ξ_1) in dB has been calculated for QPSK, 16-QAM and 64-QAM with and without interference compensation. This has been done for several different codes ranging from $v = 2$ to $v = 8$. With (10) and (13) we have evaluated an upper bound on the average bit error probability for QPSK with rate 1/2 and 3/4 coding for systems without interference cancellation and rate 1/2 coding for systems with cancellation. The results are shown in Figs. 4–6 where we have plotted the average bit error probability versus E_b/N_0. Fig. 4 shows that the coding gain at, e.g., 10^{-6} with interference is much larger than the coding gain in Gaussian noise alone. For the $v = 6$ code, the coding gain for the Gaussian channel (interference level = $-\infty$ dB) is about 6 dB while it is almost 9 dB for 20 log $(\xi) = -10$ dB and about 13 dB for an interference level of -5 dB. It is also clear from Fig. 4 that the relative gain by increasing the code memory from $v = 2$ to $v = 6$ is substantially larger for channels with interference than in Gaussian noise only.

Fig. 4 shows QPSK with a cross-polarized channel and rate 1/2 coding. This system has the same bandwidth as QPSK without coding and no cross-polarized channel. The latter system has of course no cross-coupled interference. When these two systems are compared, we can conclude that a net coding gain in E_b/N_0 at equal bandwidth and data rate is obtained with the coded system for interference levels up to a certain level, see Fig. 4 where the coded curves now should be compared to the ideal uncoded QPSK curve without interference. For a bit error probability of 10^{-6}, this level is about -5 dB for $v = 6$ and a little larger than -10 dB for $v = 2$. Fig. 5 shows performance results for QPSK with rate 3/4 coding. In this case we can observe that both power efficiency and bandwidth efficiency can be gained with QPSK and rate 3/4 coding in a dual-polarized channel than with QPSK alone in a single-polarization transmission channel. For uncoded QPSK, see Fig. 4. We show in Fig. 6 the performance for coded, rate 1/2 QPSK schemes with cancellation. The coding gains in interference are now somewhat smaller than those in Fig. 4 corresponding to no cancellation. Still, the coding gain in interference is larger than that in only Gaussian noise and increased code memory yields extra large coding gains in interference.

We will now compare rate 3/4 coded 16-QAM schemes with uncoded 8-AMPM. In [1] we compared dominating error event probabilities for these systems. Here we will compare the approximate average bit error probabilities. Fig. 7 shows the average bit error probability results for rate 3/4 coding and 16-QAM for two codes with two coded and two uncoded bits in each 4-bit representation of the 16-QAM signal points. The two codes used are the $v = 2$ code (shown with 16-QAM in Fig. 2) and the $v = 5$ code (shown with 64-QAM in Fig. 3). From the results in Fig. 7 we can see that the performance for Gaussian noise is not improved by extending the code memory from $v = 2$ to 5 while maintaining only two coded bits per signal point. The performance in both cases is given by the dominating minimum distance event, which is the parallel transition. By increasing the interference to -20 dB ($\xi = \xi_1 = \xi_2 = 0.1$), the relationship between the bounds for the $v = 2$ and $v = 5$ cases are approximately the same as for the case with no interference. The parallel transition still dominates. In [1] and above we showed that when the interference increases, the long coded error events dominate the overall error probability and the parallel transition error event contributes

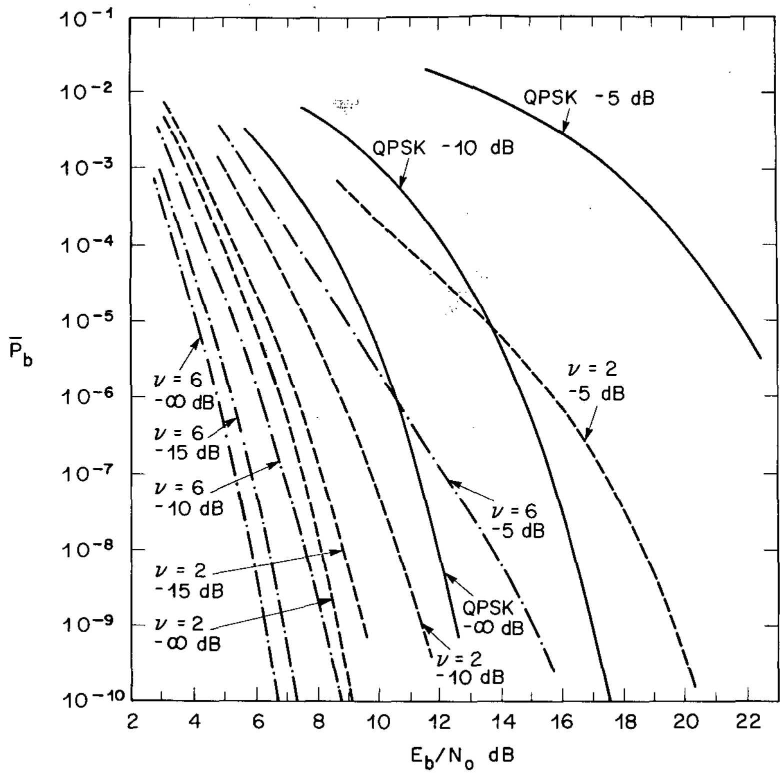

Fig. 4. Average bit error probability $\bar{P}_b$ for QPSK and coded QPSK (upper bound) for rate 1/2, $v = 2$ and $v = 6$ codes. No interference compensation is used.

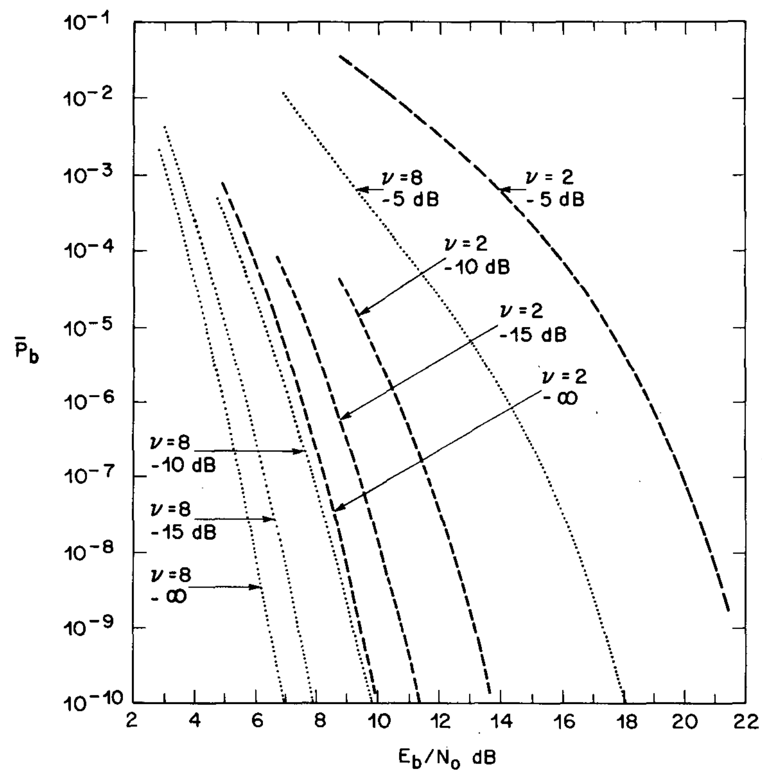

Fig. 5. Average bit error probability $\bar{P}_b$ for coded QPSK (upper bound) for rate 3/4, $v = 2$ and $v = 8$ codes. No interference compensation is used. Compare to uncoded QPSK in Fig. 4.

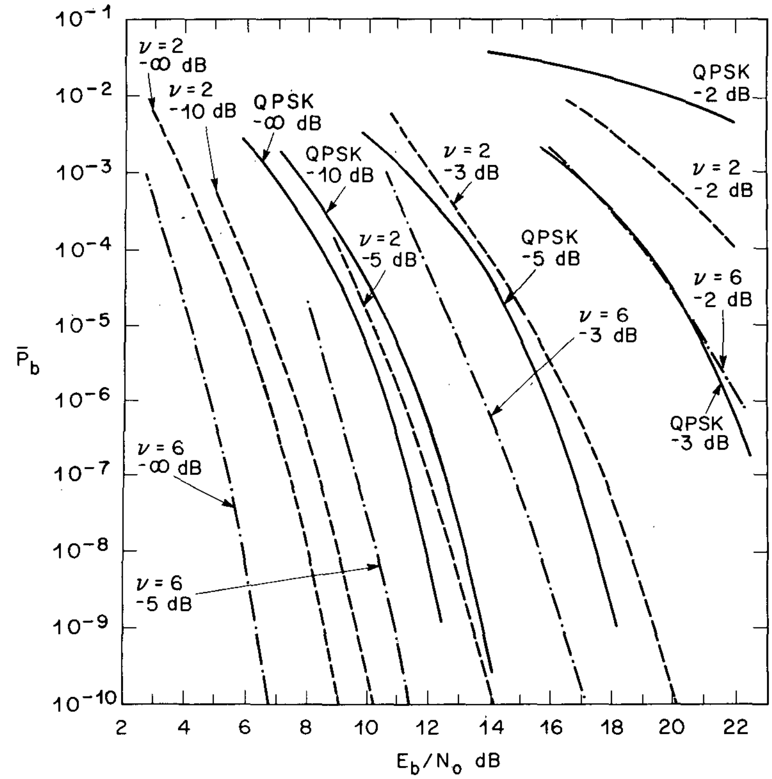

Fig. 6. Average bit error probability $\bar{P}_b$ for QPSK and coded QPSK (upper bound) for rate 1/2, $\upsilon = 2$ and $\upsilon = 6$ codes. Interference compensation is used.

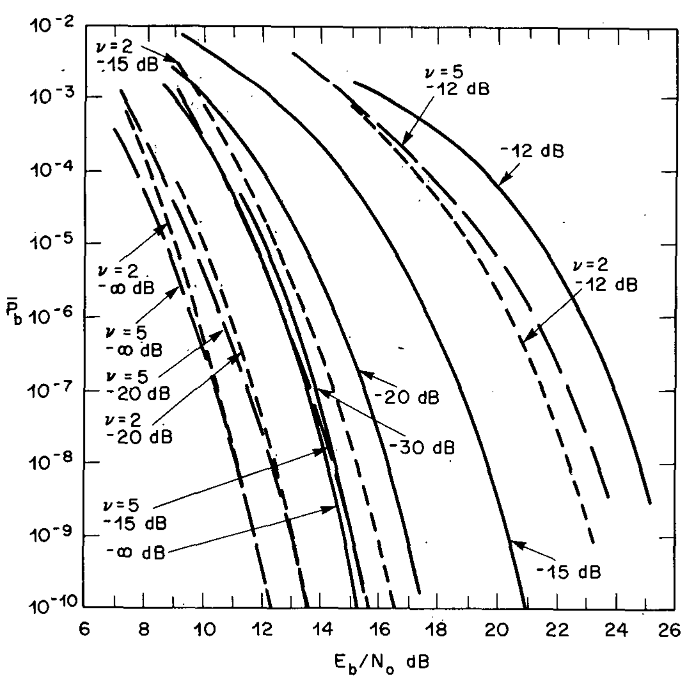

Fig. 7. Average bit error probability $\bar{P}_b$ (approximate upper bound) for rate 3/4 coded 16-QAM and for 8-AMPM (solid). No interference compensation is used.

less. At -15 dB interference, we can see this phenomenon in Fig. 7. From Section IV, we know that the bound in (19) that is used in Fig. 7 is tight for the $\upsilon = 2$ code but loose for all signal-to-noise ratios for $\upsilon = 5$. This has to do with the fact that the exact error structure for all code error events has to be known to make a precise evaluation. We used a simplified upper bound with the worst error structure. For -12 dB interference this becomes evident, where the loose upper bound on the $\upsilon = 5$ code is actually worse than the tight upper bound on the $\upsilon = 2$ case. Even with the simplified loose upper bound we can make our point, that is, the average overall bit error probability can be improved by increasing the code memory in intermediate-level interference channels, e.g., -15 dB. For comparison, Fig. 7 also shows the approximate average bit error probability for uncoded 8-AMPM (solid curves).

Fig. 8 shows the average bit error probability for coded 16-QAM with interference compensation. For this case, the simple upper bound is tight for all υ and can easily be evaluated for all codes. The improvement for the $\upsilon = 5$ case over the $\upsilon = 2$ case in Fig. 8 is obvious. Again, for low interference levels, there is no improvement since the same parallel transition error event dominates. Compare the approximate bit error probability for 8-AMPM with interference compensation (solid curves). The coding gains predicted in [1] based on dominating error event probability evaluation are confirmed by the bit error bounds in this paper. By comparing Figs. 7 and 8 we note, e.g., that the approximate required E_b/N_0 to obtain a bit error probability of 10^{-6} at an interference level of -15 dB is given by Table III. As can be expected, the coding plus compensator scheme outperforms the others. Coding alone for 16-QAM is better than compensation for 8-AMPM for weak interference. For strong interference, compensation is required.

Figs. 9 and 10 show results for 64-QAM with rate 5/6 coding and 32-AMPM without coding for the cases without

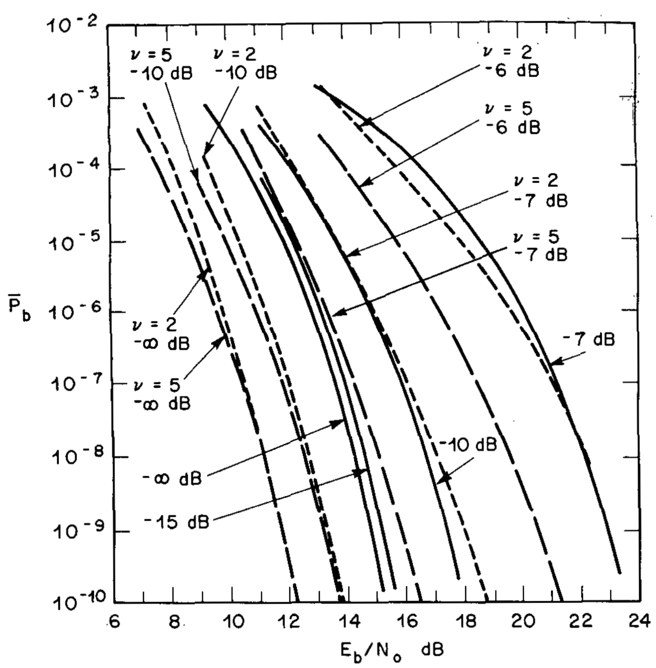

Fig. 8. Average bit error probability $\bar{P}_b$ (approximate upper bound) for rate 3/4 coded 16-QAM and for 8-AMPM (solid). Interference compensation is used.

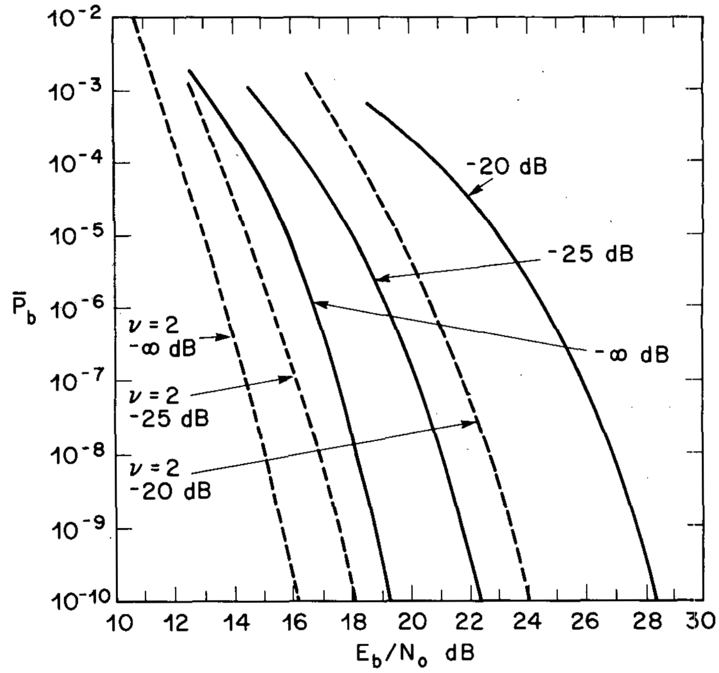

Fig. 9. Average bit error probability (approximate upper bound) $\bar{P}_b$ for rate 5/6 coded 64-QAM and for 32-AMPM (solid). No interference compensation is used.

TABLE III

COMPARISON OF 8-AMPM AND RATE 3/4 CODED 16-QAM AT $\bar{P}_b = 10^{-6}$ AND -15 dB INTERFERENCE

Scheme	Required E_b/N_0	Gain
8-AMPM	17.8 dB	—
8-AMPM with compensator	13.2 dB	4.6 dB
16-QAM $\nu = 2$ coding	12.9 dB	4.9 dB
16-QAM $\nu = 5$ coding	11.9 dB	5.9 dB
16-QAM $\nu = 2$ coding with compensator	9.2 dB	8.6 dB

and with interference compensation. The relationship of coded 64-QAM to 32-AMPM is similar to the relationship of 16 QAM to 8-AMPM. For coded 64-QAM we have used the approximate average bit error probability bounds developed in Section IV. From Fig. 10 it is observed that uncoded 32-AMPM with cancellation requires an E_b/N_0 of about 24.8 dB at $\bar{P}_b = 10^{-6}$ and an interference level of -10 dB while rate 5/6 coded 64-QAM with cancellation requires an E_b/N_0 of 19 dB for the $v = 2$ code and 17.1 dB for the $v = 5$ code. This is a gain of 5 dB with the $v = 2$ code over 32-AMPM and a gain of 7.1 dB with the $v = 5$ code. The corresponding gains in Gaussian noise are both 3 dB.

The dual-channel polarization hopping concept in Section V has been approximately analyzed for the case of QPSK with rate 1/2 coding without interference cancellation. The curves in Fig. 11 should be compared to those in Fig. 4. Note that with $\xi_2 = 0$, without polarization hopping, one coded scheme is interference free and the performance of the other corresponding to ξ_1 is given by the curves in Fig. 4. With *dual-channel polarization hopping*, both coded schemes have the performance given in Fig. 11. Note the significant improvement, especially at low bit error probability. From Figs. 4 and 11 we can observe an improvement of about 7 dB in channel signal-to-noise ratio at $\bar{P}_b = 10^{-8}$ for the worse of the two channels when a $v = 2$ code is used. This is an upper bound on the improvement for ξ_2-values in the interval $0 < \xi_2 < \xi_1$.

VII. SUMMARY AND CONCLUSIONS

Upper bounds on the average bit error probability for convolutionally coded QAM schemes in cross-coupled interference channels have been derived. We have used a truncated union bound technique and averaged the error probability over the interference by means of the method of moments. We have empirically observed that while 5 or fewer terms are enough to calculate the union bound at low values of $\bar{P}_b$ in Gaussian noise only, up to 10 terms are required for the interference channels. That is, certain long error events with a larger nominal Euclidean distance contribute almost nothing to the overall error probability for the Gaussian channel case while their contribution could be significant for the interference channel case. It is necessary to use the upperbound technique outlined in this paper if the overall bit error probability at a certain E_b/N_0 is to be evaluated. The technique in [1] based on dominating error events gives the correct trends for a high E_b/N_0 when one system is compared to another. It also gives the correct bit error probability for asymptotically high E_b/N_0's. For "reasonable" bit error probabilities like 10^{-6} and high interference levels, the upperbound method should be employed. For QPSK (4-QAM), the coding is conventional and in this case the bounds derived, are true upper bounds. For trellis coded 16-QAM and 64-QAM, the bounds are approximate, since some error events are not included. The approximations used are very good. The bounds are asymptotically tight for high channel signal-to-noise ratios.

The main conclusion in this paper is that trellis-coded QAM schemes give larger coding gains in cross-coupled interference channels than in Gaussian noise only. Furthermore, the choice of optimum code for the trellis-coded QAM scheme depends on the expected interference level. The code which is optimum for the Gaussian channel is not in general optimum for the

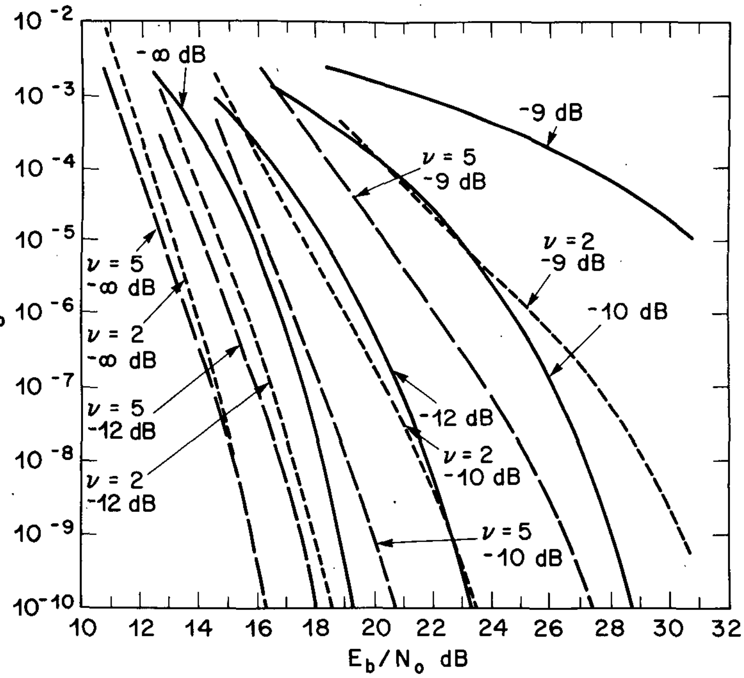

Fig. 10. Average bit error probability (approximate upper bound) $\bar{P}_b$ for rate 5/6 coded 64-QAM and for 32-AMPM (solid). Interference compensation is used.

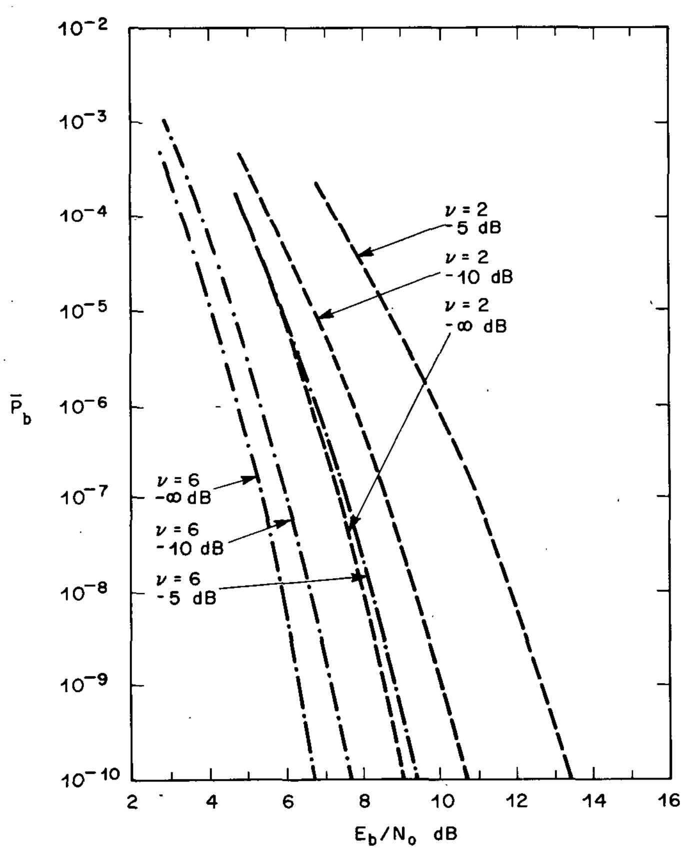

Fig. 11. Average bit error probability for QPSK, rate 1/2 coding and *dual-channel polarization hopping* for $\xi_2 = 0$. No interference compensation is used.

interference channel. For the particular schemes considered in this paper we note that rate 1/2 convolutional codes with code memory $\upsilon > 2$ applied to two of the bits in each signal point representation can be used to significantly improve performance of a QAM signal in interference. A nice extra advantage with this scheme is that the decoder is simpler than for codes where more than 2 bits per signal point are coded. The concept of *dual-channel polarization hopping* for channels with independent cross-coupling was introduced here. As an example QPSK with rate 1/2 coding was considered and a significant improvement was observed in the coding gain. For the $\upsilon = 2$ code (4 states), the improvement at $\bar{P}_b = 10^{-6}$ is about 6 dB in channel signal-to-noise ratio for the worst channel.

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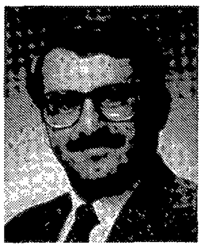

Mohsen Kavehrad (M'78-SM'86) was born on January 1, 1951. He received the B.S. degree from Tehran Polytechnic Institute, Tehran, Iran, in 1973, the M.S. degree from Worcester Polytechnic Institute, Worcester, MA, in 1975, and the Ph.D. degree from the Polytechnic Institute of New York, Brooklyn, in 1977.

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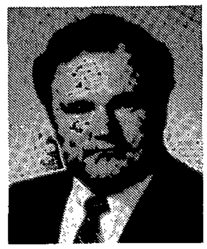

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