

# Transmission Channel Model and Capacity of Overhead Multi-conductor Medium-Voltage Power-lines for Broadband Communications

P. Amirshahi and M. Kavehrad (FIEEE)

The Pennsylvania State University,  
Department of Electrical Engineering,  
Center for Information & Communications Technology Research (CICTR)  
University Park, PA 16802  
Phone: (814) 865-7179  
E-Mail: mkavehrad@psu.edu

**Abstract** – A channel model suitable for multi-wire overhead medium-voltage lines is proposed. The model, incorporating ground admittance, is more appropriate for broadband communications. This model is then employed in order to evaluate the multipath channel impulse response and the associated transmission capacity limit in actual overhead medium-voltage power distribution networks for broadband power-line communications applications.

**Keywords** - channel model, impulse response, power-line communications, ground admittance, medium voltage, capacity.

## I. INTRODUCTION

Fast Internet access is growing from a convenience into a necessity in all aspects of our daily lives. Unfortunately, this has been held back by the high expenses of wiring infrastructure essential to deliver such high-speed internet access especially to private homes, small offices and rural areas, where the installation of any kind of new wires tilts the scales of the economic feasibility to a non-profitable state. This problem is known as the “last mile problem” which has been an active area of research throughout research community.

The lines in power delivery network can be categorized based on several criteria. Depending on line voltage, HV (high voltage), MV (medium voltage), and LV (low voltage) are typically defined. Within a distribution grid, depending on the topological configuration, either overhead lines or underground cables are used.

Overhead MV lines differ considerable in

structure and physical properties compared to other wire-lines as twisted-pair, coaxial and fiber-optic cables. The power-lines in USA hang overhead at a height of ~10 meters above ground, for more than 85% of locations, simply because over ground wiring is 10 times cheaper than underground. Typically, 4 wires, three phases and a neutral (sometimes a grounded neutral) with ~ 1 meter spacing between wires, are used over the earth. Wires are made of aluminum with 70% of standard annealed copper conductivity.

Originally designed for power delivery rather than signal transmission, power-line has many non-ideal properties as a communications medium. Impedance mismatches at joints cause reflections that generate conditions similar to those created by multipath fading in wireless communications. While there have been lots of research efforts to characterize the European ground cables, there has not been available a proper theoretical model for multi-conductor overhead MV lines, a typical situation in USA.

As an extension of work by D’Amore et al [1], in this paper, a channel model suitable for multi-conductor overhead MV lines is presented. The suggested model accounting for ground admittance is more appropriate for higher frequencies than predicted by the Carson’s model [2]. The proposed model is further used to evaluate the channel impulse response and transmission capacity in actual power distribution networks.

In section II, a review of existing models is provided. In section III, for these lines, a new communications channel model is presented. In section IV, numerical performance results are

presented. Concluding remarks and references end the discussion.

## II. REVIEW OF EXISTING RESEARCH EFFORTS

### A. Single conductor over lossy earth

Historically, “a thin wire over earth” problem was of interest to researchers since early 20<sup>th</sup> century because of its application in power transmission and telephone communications. These systems operate at very low frequencies. At these frequencies, height of wire is a small fraction of wavelength and all the coupled energy into the wire propagates in quasi-TEM mode. Thus, the early works in this field were focused on finding the distribution characteristics of this propagation mode in transmission line.

Carson reported the earliest solution for this problem in 1926 [2]. In his work, he calculated values for distribution parameters of a quasi-TEM mode in a transmission line. In doing so, he made some assumptions. These assumptions restrict the solution to very low frequencies and/or perfectly conducting earth.

To find the exact solution for this problem at high frequencies with lossy ground return, we need to derive modal equations. Kikuchi [3-4] in 1956 derived an exact modal equation for very thin wires above the earth. In this work, he used quasi-static and asymptotic expansion of the exact modal equations to investigate the transition from quasi-TEM to surface wave propagation. Carson’s method is essentially a low frequency approximation of transmission line mode. On the contrary, Kikuchi’s result is associated with the entire frequency spectrum of the same mode. Kikuchi showed, experimentally and theoretically, that as frequency increases, the transmission line quasi-TEM mode reverts to a TM mode. According to Kikuchi, as frequency increases there exists a high field concentration around the wire and large longitudinal displacement currents that act as return currents over air, thus minimizing role of the earth as return current path. Therefore, after a certain frequency, the path loss of transmission line diminishes by increasing the

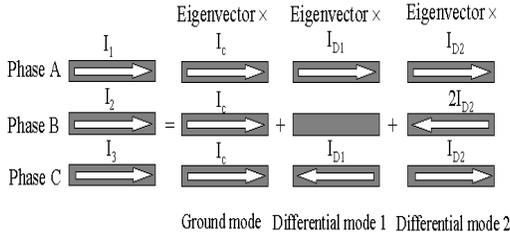
frequency. In 1972, Wait [5] extended Kikuchi’s work and could derive and solve exact modal equation for a thin wire above the earth.

### B. Analysis of Multi-conductor Transmission Lines

Analysis of transmission lines consisting of two parallel conductors has been a well-understood topic. This understanding can be further extended into matrix notations to cover multi-conductor transmission lines (MTL), involving more than 2 conductors [6]. For an MTL with  $(n+1)$  conductors placed parallel to the  $x$ -axis, there are  $n$  forward- and  $n$  reverse-traveling waves with respective velocities. These waves can be described by a coupled set of  $2n$ , first-order, matrix partial differential equations which relate the line voltage  $V_i(x, t)$ ,  $i=1, 2, \dots, n$ , and line current  $I_i(z, t)$ ,  $i=1, 2, \dots, n$ . Each pair of forward- and reverse-traveling waves is referred to as a mode. For example, in the case involving 3 conductors and a ground return, we can define 3 modes as shown in Figure-1 [7]. Using these independent modes, we can decompose currents  $I_1$  through  $I_3$  as a linear combination of 3 modal currents. Common mode (also called ground mode) is characterized by the highest attenuation among the modes, and is propagation through 3 phases and a return via the earth. Involving signal propagation and return only through wires, differential modes (also called aerial modes) 1 and 2 show a somewhat lower attenuation than common mode. While the common mode current  $I_c$  is the same in magnitude and in direction for 3 lines, the differential mode currents  $I_{D1}$  and  $I_{D2}$  are the same in magnitude but differ in direction for 3 lines.

In BPL, depending on the way signal is coupled to the lines, either wire-to-wire (WTW) or wire-to-ground (WTG) injection is feasible. For WTW injections, differential modes are mostly excited. For a WTG injection, in the case of coupling to the middle phase, common mode and differential mode 2 are excited. Generally, these modes are not orthogonal unless the wavelength of electromagnetic wave inside the conductors is a

small fraction of the height of wires and the spacing between the wires is a small fraction of wavelength [8]. This condition is satisfied for practical MV power-line systems up to 100 MHz. Beyond this frequency, the discrete modes lose orthogonality and continuous modes start to appear.



**Figure-1 Modes of three-phase power-lines**

Basically, this problem is solved by solving the so-called curl-Maxwell equations and satisfying the boundary conditions on each and every wire [8-9]. By doing so, the result for transmission constant is the answer to matrix equations with Bessel and Sommerfeld integrals.

Actually, taking a number of steps can solve for  $n$  line voltages and  $n$  line currents, describing MTL. First, *per-unit-length* parameters such as inductance, capacitance, conductance and resistance are determined for the considered line. Secondly, the MTL equations are solved in the form of a sum of  $n$  forward- and  $n$  reverse-traveling wave equations, with  $2n$  unknown coefficients. Thirdly, termination conditions such as independent voltage/current sources, load and source impedance values are incorporated in the MTL equations in order to determine the  $2n$  unknown coefficients [6].

As stated earlier, the first step in solving the MTL equations is to obtain *per-unit-length* parameters for the conductors. For this, Carson [2] suggested incorporating ground impedance. However, this model, without considering the ground admittance, is only suitable over low frequency values and/or under good conductive ground plane conditions.

Next, as an effort to find a new ground return path model for higher frequencies and/or under poor ground conductivity conditions, a new

procedure was suggested. This methodology by D'Amore et al [1] incorporates *per-unit-length* series-impedance and shunt-admittance matrices, using the curl-Maxwell field equations.

### III. BPL NETWORK CHANNEL MODEL AND CAPACITY

Frequency response,  $H(f)$ , of a matched transmission line can be expressed by means of a propagation constant,  $\gamma$ . In [10], voltage along the conductor at a distance  $l$  from the source,  $V(l)$ , is obtained by:

$$V(l) = H(f)V(0) \quad (1)$$

$$H(f) = e^{-\gamma(f)l} = e^{-\alpha(f)l} e^{-j\beta(f)l} \quad (2)$$

where  $V(0)$  is the voltage at the source. By having the propagation constant, one may easily find a transfer function for power-line wire at a desired point on the conductor. As discussed later, each mode of coupling has a different propagation constant. Hence, there is a different frequency response for each mode.

Part of a propagating signal reflects back to transmitter at branch junctions due to impedance mismatch and the remainder travels through [10]. Reflection coefficient is defined for each node as ratio of reflected signal power to total received signal power at the node. In the same way, transmission coefficient is defined as the ratio of transferred signal power to the total received signal power at the node. Obviously, reflection and transmission coefficients are equal or less than unity and the sum of all transmission and reflection coefficients at each node is unity.

Signal propagation does not take place along a direct path from a transmitter to a receiver in a power-line network. Additional paths (echoes) also exist due to reflection at the network junctions. This creates a multipath scenario with frequency selectivity, similar to a radio channel. Each arrived path at a receiver is weighted by a factor,  $g$ , which is the product of reflection and transmission coefficients of nodes along the path. As reflection and transmission coefficients are equal or less than one, the weighting factors are

equal or less than unity, as well.

With these weighting coefficients, we may express the network as a summation of multiple paths with different length and weighting factors. The propagation along a wire follows (2), so one can easily express the multipath network channel model as:

$$H(f) = \sum_{i=1}^N g_i e^{-\alpha(f)d_i} e^{-j\beta(f)d_i} \quad (3)$$

where  $N$  is the number of significant arrived paths at the receiver,  $d_i$  is the length of  $i^{\text{th}}$  path and  $g_i$  is the weighting factor of the  $i^{\text{th}}$  path. This formulation is basically similar to what has been mentioned in [11], however, with a model for propagation constant that is appropriate for overhead MV power-lines, rather than underground cables in Europe.

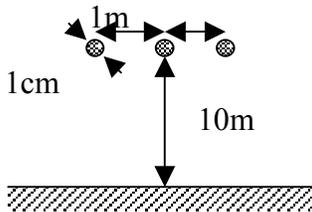
By applying water-filling [12] in spectral domain, we can express channel capacity as:

$$c = \int_{-\infty}^{\infty} \frac{1}{2} \log_2 \left[ 1 + \frac{\left( p - \frac{N_0(f)}{2|H(f)|^2} \right)^+}{\frac{N_0(f)}{2|H(f)|^2}} \right] df \quad (4)$$

where  $p$  is the signal power at the specific frequency and is chosen such that

$$\int_{-\infty}^{\infty} \left( p - \frac{N_0(f)}{2|H(f)|^2} \right)^+ df = P. \quad \text{The notation } [X]^+$$

means  $\text{Max}\{X, 0\}$  and  $P$  is the average transmitted power. In (37),  $N_0(f)$  is the noise spectral density in the system.

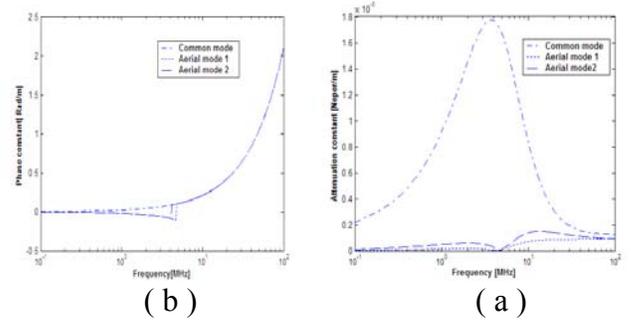


**Figure- 2 Geometrical configuration of power-line model used in numerical evaluations.**

#### IV. NUMERICAL COMPUTATION RESULTS

For numerical computation purposes, we used a three-wire configuration depicted in Figure- 2. The ground plane is characterized by relative permittivity of  $\epsilon_g = 13$  and conductivity of  $\sigma_g = 5$  mS/m.

The frequency spectra of the real and imaginary parts of three propagation constants are computed by using the method described earlier and the results are represented in Figure-3 a and -b.

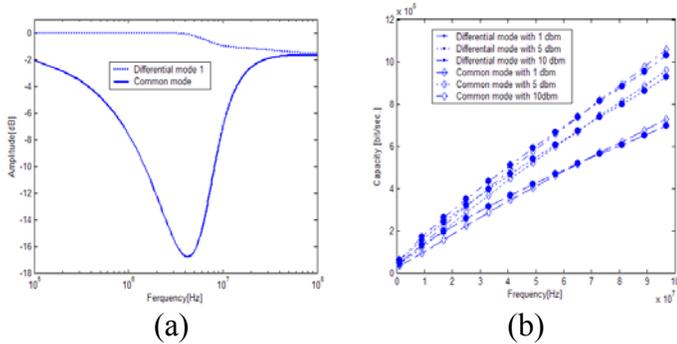


**Figure- 3. Frequency spectra of (a) Attenuation constants, and (b) Phase constants of MTL system shown in Figure- 2.**

Phase constants overlap on almost the entire frequency range. On the contrary, attenuation constants show different behavior and values. Common mode shows higher attenuation over the frequency range and the attenuation factors for the two aerial modes are close to one another. Common mode attenuation factor increases up to some frequency and decays beyond. This incident is due to resonance phenomenon in ground medium, initially inductive and by increasing frequency it exhibits a capacitive behavior. The aerial mode is just involved with wires and loss in this mode originates from loss in wires and as it is shown in Figure-3a, loss in this mode increases monotonically with frequency. This behavior is very well understood and the detailed reason is given in [13].

Figure- 4a represents frequency response of a matched transmission channel over a 1 km span MTL system. As the system is matched, signal does not reflect at the receiver-end and signal path is one straight point-to-point path. In this case, the

only loss comes from MTL path loss. Figure- 4a depicts frequency response for two coupling methods: common mode and differential mode 1. Common mode exhibits more loss than differential mode, especially at low frequencies. As frequency increases, losses of the two configurations become comparable. Also, one may notice that both systems show a very low loss at high frequencies over a 1 km repeater span. The fact that MV overhead power-lines resemble a low loss transmission system shows promise for data delivery at high rates.

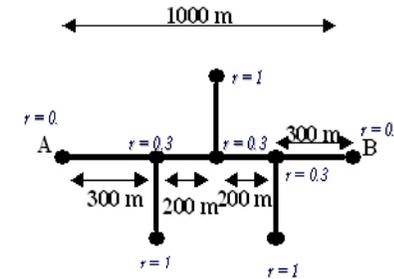


**Figure-4. (a) Frequency response of matched MTL transmission over 1 km for differential and common mode coupling (b) Corresponding capacity values for different coupling methods and transmit power levels.**

Figure-4b illustrates the water filling channel capacity limits of (4) for matched transmission system with a 1 km repeater span at different transmitted power levels. For evaluating channel capacity, we chose a uniform -105 dBm/Hz as a representative of average noise spectral density height. Referring to [14], this value is a conservative average estimate of practical background noise level for MV power-lines in Korea. It is interesting to see both differential and common modes coupling systems show almost the same capacity characteristics, especially at high frequencies. This is due the fact that both systems are approaching the same loss level at higher frequencies.

According to Figure-4b, with an ideal matched MTL system, over 50 MHz of channel band, we can deliver almost 600 Mbps by launching 10dBm transmit power. In reality, this low loss nature of MTL systems degrades extensively by several impairments.

Over an actual power-line network, there always exist several branches and junctions between a transmitter and a receiver. These branches cause nulls in transmission channel frequency response due to multipath. To investigate this phenomenon, we simulated the complex network shown in Figure-5. In this network we have three branches between transmitter and receiver. Each end of these branches is an open-circuit, so reflection factor at each end is one. Also, we have assumed that transmitter or receiver impedance is matched to that of the line. For simplicity, we chose the reflection factor for all the middle junctions to be 0.3.



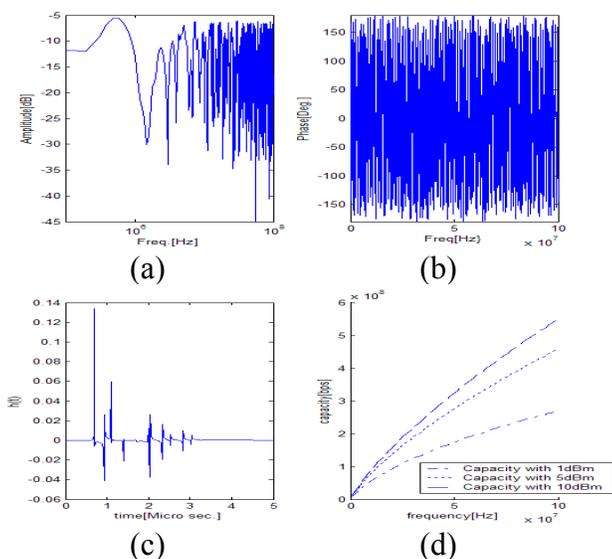
**Figure- 5 The simulated complex network.**

Figures-6a and b show amplitude and phase of complex network frequency response. Reflections create deep nulls in the frequency response. Simulation shows there are 12 dominant paths and from Figure-6c, 12 pulses with different arrival times are distinguished. Delay spread in this network is almost 3 microseconds. Figure-6d is the illustration of the channel capacity limits for this complex network. The average capacity in this network with a 10 dBm launched transmit power level at 50 MHz band is about 300 Mbps Obviously, the junctions and branches between transmitter and receiver degrade the system performance extensively compared to the ideal point-to-point case.

## V. CONCLUSIONS

This research dealt with examining MV overhead power-lines as a communications medium for broadband transmissions. Available models for overhead power-lines were not suitable for high frequencies with lossy ground return. D'Amore et al in [1] have proposed a model for multi-wires

over ground, which is more suitable for application of BPL systems using overhead MV lines. Based on this model, we developed a new channel transfer characteristic function model. Our simulations show ideal overhead power-lines exhibit a low loss with a capacity limit of about 1Gbps over a 1 km repeater span, if 10 dBm transmit power and 100 MHz of channel bandwidth are available. Junctions and branches in power-line network cause signals to reflect and produce a multipath channel. This causes degradation in power-line system performance and decreases channel capacity. Removing discontinuities by adaptive impedance matching [15] on these lines can enhance line data handling capacity. Use of differential aerial modes along with symmetric loads on lines can potentially reduce interference.



**Figure-6** Frequency response of complex network shown in Figure-5: (a) Amplitude, (b) Phase, (c) Channel impulse response and (d) Lines transmission capacity bounds.

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